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Computer-Aided Reconstruction and New Matches in the Forma Urbis Romae

David Koller and Marc Levoy, Stanford University

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Introduction

The Stanford Digital Forma Urbis Project[1], under the aegis of the Sovraintendenza ai Beni Culturali del Comune di Roma, has been applying digital technologies to further archaeological study of the Severan Marble Plan of Rome. We have digitized the shape and surface of all the extant marble map fragments using high resolution laser range scanners and color cameras, and we have assembled this data into a collection of three-dimensional computer models and color photographs. Along with additional supporting archaeological documentation, these materials have been organized into a publicly available online database for use as a tool for scholarship on the Plan.

Aside from the intrinsic value of the public database, the digital representations of the map fragments enable new kinds of research and analytical study of the Marble Plan. In particular, we are using advanced computer searching and matching algorithms to aid in reconstruction of the Plan through virtual reassembly of the fragments. Traditional scholarship has also focused on joining the surviving fragments and reconstructing the map, but progress has been painstaking and slow, in part due to the difficulty of accessing and working with hundreds of unwieldy marble fragments. However, computer programs that operate on digital models of the fragments can rapidly and systematically consider many thousands of possible fragment positions and combinations. Such computer-aided fragment reassembly procedures have been previously applied to archaeological problems, and represent an active area of research in computer science[2].

In this paper, we describe our efforts to apply computer-aided reconstruction algorithms to find new matches and positionings among the fragments of the Forma Urbis Romae. First, we review the attributes of the fragments that may be useful clues for automated reconstruction. Then, we describe several different specific methods that we have developed which make use of geometric computation capabilities and digital fragment

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[1] The Project website address is *http://formaurbis.stanford.edu*.

[2] Examples include R.W. SMITH, *Computer helps scholars re-create an Egyptian temple*, in *National Geographic Magazine*, November, 1970, pp. 634-655, and H.C.G. LEITAO, J. STOLFI, *A Multiscale Method for the Reassembly of Two-Dimensional Fragmented Objects*, in *IEEE Transactions on Pattern Analysis and Machine Intelligence*, 24.9, 2002, pp. 1239-1251.

representations to suggest new matches. These methods are illustrated with a number of new proposed fragment joins and placements that have been generated from our computer-aided reconstruction process.

Reconstruction Clues

The remains of the Plan include a number of properties that are potentially useful as clues for automated fragment reconstruction. The most obvious is the inscribed map topography on the marble surface, which has been the primary source of information for prior reconstruction scholarship[3]. Another strong clue is the fracture shape along the fragment edges; adjacent fragments with edge geometry that is not substantially eroded should fit together like pieces of a jigsaw puzzle.

Fragments that originated from the same marble slab typically share some common characteristics. The thicknesses of adjacent fragments from the same slab are usually similar, and the thickness gradient can be useful for determining placement and orientation of fragments within a slab. The marble veining direction of fragments within a slab is also normally consistent. Some slabs had rough back surfaces while others had smooth surfaces, contributing another property useful for the grouping and matching of fragments. The presence of straight slab edges, clamp holes, and wedge holes (*tasselli*) on the fragments can help join fragments together, as well as providing information about the orientation and position of fragments on the original map wall[4].

The fragment attributes described above are all appropriate for use in our computer algorithms due to their geometric nature. We are easily able to model the position and nature of these features in our digital representations of the fragments, and our algorithms utilize these geometric constraints to search for and find potential fragment matches. There are many other properties of the fragments that are useful in reconstruction that we have not exploited, such as geological characteristics of the marble (color, texture, etc.), the style of incisions (*ductus*), and correlation of the depicted topography to known excavated architecture.

Computer-Aided Reconstruction Methods

We have developed and experimented with several different computer algorithms for automating reconstruction of the Forma Urbis fragments. Though the various methods employ different combinations of particular reconstruction clues as their primary inputs, all of the techniques operate by searching a huge space of possible fragment correspondences and positions, and seeking the arrangements which best satisfy the specified set of geometric constraints. These geometric constraints are used by the algorithms both to guide the search process in an efficient manner, as well as to evaluate the quality of proposed positions. Each automated technique that we have implemented outputs listings of proposed fragment matches and specific relative positions, scoring each proposal according to its quality and sorting the output lists in ranked order. Archaeologists can then review the top scoring

[3] A major reconstruction effort occurred in 1741-1742 when many of the known fragments were exhibited in wooden frames in the Capitoline Museums. In the 20th century, CARETTONI-COLINI-COZZA-GATTI 1960 and RODRÍGUEZ-ALMEIDA 1981 reinvigorated the reconstruction process by using additional constraints such as marble veining and clamp holes.

[4] The discovery of the relationship between *tasselli* on the back of fragments and mortar patches on the aula wall is described in E. RODRÍGUEZ-ALMEIDA, *Forma Urbis marmorea: Nuovi elementi di analisi e nuove ipostesi di lavoro*, in *Mélanges de l'Ecole Française de Rome, Antiquité*, 89.1, 1977, pp. 219-256.

matches proposed by the computer, and apply further higher-level reasoning beyond the simple geometric constraints; they may consider excavational evidence, ancient literary references, or other Roman topographical knowledge that is not digitally encoded and taken into account by the algorithms.

In the sections below, we describe the particular algorithmic approaches in more detail, and demonstrate them with examples of new proposed fragment matches and positions suggested by our computer-aided methods.

Boundary Incision Matching

The automated boundary incision matching technique searches for incised topography that corresponds across the boundaries between two candidate adjacent fragments. To prepare our digital fragment representations for input to this algorithm, we manually annotated all of the incised features that leave the boundaries of each fragment, indicating their positions, directions, and feature types (such as rows of columns, *tabernae* fronts, aqueducts, text inscriptions, etc.). This was done using custom computer software that allowed us to trace the incised boundary features on a digital image of the fragment, and enter a brief code designating the feature type. The topographic feature types were selected from a hierarchy of over 150 possible labels (fig. 1). This hierarchical arrangement of feature types allowed annotations to encode varying levels of certainty about the topography depicted by particular incisions. A straight solid line leaving an otherwise blank fragment could simply be labeled as a "straight solid line" feature, whereas if enough evidence on the fragment existed to better identify the type of line (such as a straight solid line depicting the back of a row of *tabernae*), we could use a more specific feature type label deeper down in the hierarchy. We annotated approximately 12,500 total unique boundary features among all of the Forma Urbis fragments.

This collection of annotated boundary incision features is the input to the boundary incision matching algorithm. The algorithm searches through all of the annotated fragments, considering each possible pair of fragments, and then considers each reasonable alignment of annotated features between the fragment pairs. Candidate configurations of two fragments are scored based on the alignment of the corresponding feature types, with the highest scores going to those proposed positions which have the highest number of strongly similar features that are best allineated. Fragment pairings with exceptionally high scores are output in a ranked listing for further review by an archaeological expert. Tentatively matched pairs of fragments can be considered as a single grouping in a specific alignment, and then used as input to further iterations of the algorithm; this technique allows for finding clusters of matches involving more than only two fragments.

Running this boundary incision matching algorithm on the annotated Forma Urbis fragments has so far been our most successful approach for finding new fragment matches. By reviewing the output lists of the top scoring proposed joins, we are able to efficiently discover numerous matches of considerable likelihood. Several of these new proposed fragment matches are described below.

Fragments fn23, 28a, 34b

Fragment fn23 is one of the fragments discovered in the 1999 Templum Pacis excavations and digitized at Stanford in 2001. Running the computer boundary incision matching algorithm on an annotated representation of fr. fn23 resulted in a very highly scored match with fragment fr. 28a. Annotated versions of the two fragments are shown in fig. 2.

The highest scoring position suggested by the matching algorithm situated fr. fn23 directly below fr. 28a, with five corresponding angled walls aligning across a slab edge between the two fragments. This candidate position places fn23 in the corner formed between fragments 28a and 34b (fig. 3), just north of the Via Campana-Portuensis visible on fr. 28a. This location for fragment fn23 is confirmed by the compatibility of the incised topography with that on both fragments 28a and 34b, and the constraint of the slab edge along the top of the fragment. The fragment thickness is consistent with that expected of the proposed position on the wall; fragments 34a and 34b increase from 58mm to 66mm from left to right, and fragment fn23 continues this trend with an approximate thickness of 66-67mm. The correspondence between the incisions on the actual marble fragments fn23 and 34b was verified in 2003 (fig. 4). There is a narrow gap of a couple of centimeters between the two fragments when positioned, enough clearance that the partial *tassello* hole on the back of fr. 34b is not shared with fn23.

Fragments 156, 667, 134

Another example of boundary incision matching results is demonstrated with fragments 156, 667, and 134; the annotated fragments are shown in fig. 5. The output from the matching algorithm assigned high matching scores to each pairwise combination of these 3 fragments; upon manual review, the configuration shown in fig. 6 was determined to be the most likely. E. Rodríguez-Almeida has previously confirmed the join between frs. 667 and 134 [5]. This new proposed location of fragment 156 continues the distinct row of incised *tabernae* from fragment 667, and the fourth incision on the right side of fr. 156 aligns with the fractured edge of 667 and has a corresponding incision on fr. 134. Each of the three fragments have rough back surfaces, and there is a consistent gradient of thickness decreasing from 78mm for fr. 156 down to 70mm for fr. 134.

The three fragments were examined in person in 2003 (fig. 7). No obvious geometric fit between fragments 156 and 667 was evident, although the thickness of the two fragments along the proposed interface was the same, and a two-tone banded coloration of the marble along the edges appears to correspond when the fragments are placed adjacent to one another. We are currently researching possible locations on the map for the hypothesized cluster of three fragments; the rough back surface significantly limits the set of possible slabs the fragments may belong to. Additional constraints include the slab corner and clamp hole on fr. 156, and a possible clamp hole location suggested by the unusual sloping erosion pattern on the back on fr. 667.

Fragments fn9, 351

Four vertical rows of distinct parallel incised features on fragments fn9 and 351 caused the boundary incision matching algorithm to assign a very high score for the correspondence between these two fragments (fig. 8). Due to the strictly parallel nature of the aligning features, however, the algorithm lacks enough constraints to suggest a specific distance between the two fragments. In this case, manual observation of our three-dimensional computer models of the fragments allowed us to infer that the fragments lie immediately adjacent to one another, based on the strongly corresponding shape between

[5] RODRÍGUEZ-ALMEIDA 1992, pp. 62-63.

the fractured edges along the proposed interface (fig. 9). Examination of the actual marble fragments in 2003 confirmed that the fragments mated together with a lock of the marble surfaces. Further evidence of the match is evident in the spacings of the rooms, dashed line, and row of T-shaped piers that traverse the two fragments, including the recurrence of stairways in every third room along the right side of the central street.

The topography depicted on the combined fragments fn9 and 351 is suggestive of the architecture along the sides of the Circus Maximus, as shown on other fragments of the Plan (cf. frs. 8b-g)[6]. Making this assumption, a number of possible positions on the map can be considered for frs. fn9/fn351 (a-d in fig. 10). One such position would be in the slab containing fragments 8a-h, on the upper left side of the Circus as depicted on the Plan (fig. 10(a)). However, to fit above the horizontal slab boundary along the bottom of this slab, frs. fn9/351 would have to be positioned almost directly beneath frs. 8fg, which does not appear very congruous when observing the architecture between frs. 8fg and 351 (although the fragment thicknesses are similar). Additionally, there is concern that the marble veining direction of the fragment groups may be incompatible. Although not conclusive, there is visual evidence of a possible wide gray marble vein on the left of the top surface of fr. fn9, oriented at a small angle of approximately 5 degrees from parallel with the depicted street. This would not be consistent with the distinct marble veining direction of other fragments in the slab (frs. 8a-c indicate a veining direction near horizontal across the slab). For these reasons, we think it less probable that frs. fn9 and 351 belong in this slab.

The fn9/351 fragment cluster is also not likely to lie on the upper right side of the Circus Maximus (fig. 10(b)), because a vertical slab division appears to occur within the *cavea* on the right side of the Circus. No slab edges are evident on fn9/351, even though the full width of one side of the *cavea* seems to be depicted on fn9, so the fragments seem to be impossible to place in the slab to the right of fragments 8a-h. By process of elimination, then, it seems most probable that fn9/351 should be positioned in the slab containing fr. 9 and the bottom half of the Circus Maximus (fig. 10(c,d)). Based on the similarity of the architecture on fr9/351 with that on frs. 8b-g, we propose a position for the cluster of fragments on the lower left side of the Circus rather than the right, as shown in fig. 11.

Proposed placements of fragments into previously unoccupied slabs creates the potential for a number of new follow-on fragment positionings, as slab properties such as thickness and veining direction become defined. Because fr. 9 does not have an intact back surface and thus offers little such information, an assignment of frs. fn9/351 to this slab would help determine properties for the entire slab, including a smooth back surface, thicknesses including 62-63mm, and any veining following that of fr. fn9. We are investigating a number of possible new fragment placements here that may be suitable within these hypothesized constraints, such as fr. 342, which also includes a pair of parallel incisions reminiscent of those included on other Circus fragments[7]. During close examination of fragment 342 in March, 2004, we observed an angled bedding plane in the marble similar to that in fr. 351, and would suggest a possible position and orientation for fr. 342 on the opposite side of the Circus from frs. fn9/351.

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[6] CARETTONI-COLINI-COZZA-GATTI 1960, p. 66, footnote 2, noted the similarity of fr. 351 to fragments 8d-g.

[7] CARETTONI-COLINI-COZZA-GATTI 1960, p. 66, footnote 2, mentioned fr. 342 in the context of the Circus Maximus fragments; their argument for exclusion should be revisited with the new evidence from fragments fn9 and 351.

Fragments 28a, 150

Figure 2 shows the boundary incision annotations for fr. 28a, which is located on the right bank of the Tiber River. Note the cluster of three boundary features intersecting the middle of the left slab edge of the fragment, representing a row of columns in front of a row of rooms. The boundary incision matching algorithm detected a strong match with similar aligning features on fr. 150, including the row of columns and the front wall of the row of rooms, with both of these linear features making an identical angle to the slab edges present on frs. 28a and 150. The two fragments in their proposed relative positioning are shown in fig. 12. The open space and angling row of columns depicted on fr. 150 extends and perhaps marks a corner of the interior of the large open structure portrayed on the upper left quadrant of fr. 28a, lying between the River and the Via Campana-Portuensis.

Further evidence confirming this match across slab boundaries is observable in the details of the incisions. The spacing of the columns appears uniform if we assume that one column is missing from the eroded portion in between the two fragment surfaces. The columns on both fragments are centered directly between the room openings on the fragments, and the style of the incisions appears the same. Furthermore, the thickness, veining direction, and rough back surface of fr. 150 is consistent with that expected for fragments placed in the slab to the left of fr. 28a.

Fragments 92, 138

Another highly scored match found by the boundary incision matching algorithm involves fragments 92 and 138 (fig. 13). In this case, the algorithm was able to align 3 pairs of incised linear features, and additionally was constrained by the presence of slab edges on both fragments. When both candidate fragments being considered by the matching algorithm include slab edges, the possible relative orientations of the two fragments are limited to those with the slab edges either exactly parallel or perpendicular to one another.

The street bisecting the two clusters of structures on fr. 92 aligns with that on fr. 138b, and the vertical line on the far right side of fr. 92 matches the next street over to the right, which is seen on fr. 138a. The angle of the top row of *tabernae* on fr. 92 relative to the slab edge is the same as the large street on fr. 138b, and the *ductus* of the incisions on both fragments appears very similar. Both frs. 92 and 138 have rough back surfaces, and the fragment thicknesses make a consistent gradient in the proposed positions (fr. 92 has a thickness of 63mm along the top edge, increasing towards the bottom, whereas fr. 138 continues this trend ranging from 68mm on the top to 74mm on the bottom). No marble veining information is available for fr. 92 to compare with that reported for fr. 138.

Fragment 138 was identified and located in Trastevere in recent work by P. L. Tucci[8], placing it in the lower left corner of the slab to the left of frs. 28 on the wall. The corresponding position for fr. 92, then, is along the top edge of this slab; by referencing the slab height from frs. 28, we can determine the gap between frs. 92 and 138 to be a few centimeters in extent. The clamp hole on the edge of fr. 92 would correspond with clamp hole C8-16 measured by L. Cozza[9]. Consistent with this positioning (fig. 14), the incised topographical features reflect those of a riverfront fragment. The row of dots in front of the *tabernae* are reminiscent of those on fr. 27f, and the warehouse with double stairways around

[8] P.L. TUCCI, *Eight fragments of the Marble Plan of Rome shedding new light on the Transtiberim*, in *PBSR*, 59, 2004 (forthcoming).
[9] Carettoni-Colini-Cozza-Gatti 1960, pp. 177-189, tav. LXI.

an entrance opening onto the river are very similar to fr. 28a[10]. The back wall of the riverfront *tabernae* on fr. 92 stops short of the fracture boundary on the left of the fragment, possibly establishing the position of a riverfront entrance to the structure on the left, directly opposite from the stairway entrance to the corresponding space on fr. 138b. The positioning of fr. 92 in this slab seems to depict an alignment of this section of the Tiber River making an angle of several degrees from horizontal on the wall; this is somewhat different than the course of the Tiber as previously depicted in reconstructions of the Plan.

Fragments 330, 354

Boundary incision and veining direction constraints were employed in the suggested match output for fragments 330 and 354 (fig. 15). Although only two parallel incised features delimiting a street aligned in the computer algorithm search, the additional third constraint of the marble veining direction limited the relative orientation of these two fragments to only two possibilities and helped to produce a relatively high matching score.

These three matching features alone were not sufficient to determine the specific offset distance between the two fragments. However, as in the previous case of fragments fn9 and 351, manual observation of our three-dimensional computer models allowed us to confirm that the fragments should join immediately adjacent to one another. The computer models (fig. 16) showed that both fragments exhibit unusual smooth, flat areas in their fractured edge surfaces along the proposed interface between the fragments. We were able to verify this join with the actual marble fragments in March, 2004 (fig. 17).

Rodríguez-Almeida previously joined fragments 284 and 330 together[11], resulting in the complete floorplan of an *insula*. The addition of fr. 354 to this cluster of fragments extends the depicted architecture another block, including another apparent street intersection. A number of features are present that should help to locate these fragments on the Plan, including a slab corner, a clamp hole, and a *tassello* (fig. 18). Rodríguez-Almeida noted the similarity between the depicted *insula* and one excavated underneath the Palazzo Piombino along the Via Lata, and we are investigating a number of alternative locations to position the fragments, such as along the Via Campana-Portuensis beneath frs. 27.

Fragments 37Ail, 576

Another proposed fragment match suggested by the boundary incision matching process involves fragments 37Ail and 576 (fig. 19). Four sets of incised features aligned well across the horizontal slab boundary between the two candidates; three of them lie along the sides of the parallel streets, and the fourth appears as a half-eroded incision on the lower-left corner of fr. 576, perpendicular to the street features and aligning with the incision in the upper-left of fr. 37Ai. Though this proposed match is far from certain, other similarities between the two fragments deserve its consideration. The style of the incisions on fr. 576 is very much like that of fr. 37A, and the depiction of the architecture, such as the stairways, is similar.

Rodríguez-Almeida previously placed the 37A fragment group in Trastevere, along the right edge of a horizontally oriented slab[12]. Because fr. 576 is a slab corner, its range of possible positions on the wall is highly constrained, and the proposed match with 37A does

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[10] R.A. STACCIOLI, *Tipi di 'horrea' nella documentazione della 'Forma Urbis'*, in *Coll. Latomus*, 58.3, 1962, pp. 1432-1434, includes fragment 92 in a typological study of *horrea* depicted on the Plan.

[11] RODRÍGUEZ-ALMEIDA 1992, pp. 64-66.

[12] RODRÍGUEZ-ALMEIDA 1981, pp. 140-143.

not appear compatible with the expected slab boundary locations. If positioned directly above fr. 37Ail in Rodríguez-Almeida's wall placement, fr. 576 would not be coincident with a slab corner; in fact, the closest such position would require a shift of fr. 37A to the left by almost 35 cm on the wall. The resolution of this dilemma requires further study, and is one reason to regard this proposed match with skepticism.

Fragments 141, 194

One more example of suggested matches from the boundary incision matching algorithm is fragments 141 and 194. Their proposed alignment, again across a slab boundary, is shown in fig. 20. The slab boundary is a valuable geometric constraint for the automated feature matching process, as it reduces the degrees of freedom permitted in the relative positioning of the fragments, thus greatly decreasing the number of possible combinations of fragment configurations that must be evaluated. In this case, 5 pairs of corresponding incised linear features angling across the slab boundary resulted in a high matching score. Two of the matching sets of incisions were annotated as the front of a row of rooms, and another two as the back of a row of rooms; the fifth set of matching incisions, less definitive than the others, appears as if it may represent the border of a rectangular space.

Though we have not yet researched potential placements on the wall for this hypothetical new fragment group, it contains a number of constraining elements useful for its possible positioning. Fragment 141 has a rough back surface and should fit into the corner of a slab, whereas fragment 194 has a smooth back and is only 38mm thick, so it must originate from an unusually thin slab.

Wall Feature Matching

The boundary incision matching technique described above is used primarily to find joins between pairs of adjacent fragments. Alternatively, wall feature matching is another algorithm that we have developed to position fragments in specific locations on the map. The wall feature matching algorithm searches for correspondences between structural features of the fragments (slab edges, clamp holes, and *tasselli*) and matching remnants of these features on the aula wall. We have digitized the locations of the intact clamp holes and masonry patches on the wall, based on the wall measurements collected by L. Cozza[13] (fig. 21). Additionally, as part of our boundary incision annotation procedure, we digitized the locations of any clamp holes, *tasselli*, and slab edges for each fragment.

For each fragment given as input, the wall feature matching algorithm searches all of the possible positions on the wall, assigning a score for each valid position based on the quality of the geometric correspondence between the fragment's clamp hole locations and the pattern of clamp holes on the wall. If a *tassello* is present on the fragment, its location is required to lie within a masonry patch region of the wall. Valid fragment positions are further constrained such that slab edges must be in either a vertical or horizontal orientation. The top-scoring fragment positions output by the computer algorithm are manually checked to verify their suitability in light of considerations beyond the simple geometric constraints of the wall features.

The applicability of this wall feature matching method is somewhat limited by the scarcity of the surviving constraints on both the fragments and the aula wall. Few of the

[13] CARETTONI-COLINI-COZZA-GATTI 1960, pp. 177-189, tav. LXI.

remaining unidentified fragments have *tasselli*, slab edges, or multiple clamp holes as required for input to the matching algorithm. Also, many areas of the wall have been disturbed since antiquity, so our knowledge of the clamp hole and masonry patch locations is very incomplete. Finally, our simple model of requirements for clamp hole, *tassello*, and masonry patch correspondences may be invalid in some cases, and the algorithm is highly dependent on having accurate input data for these features.

Nevertheless, we are running the wall feature matching algorithm on those fragments and fragment groups for which it is plausible. The example below illustrates one such result from this process, and demonstrates the value of having as many constraining features as possible.

Fragments 421, 475

Fragment 421 exhibits a long slab edge with two clamp holes, in addition to a *tassello* (fig. 22); these three constraints are enough to be appropriate for input to the wall feature matching procedure. When the algorithm is executed, the database of wall features is searched for an arrangement of two clamp holes matching the fragment's 35 cm horizontal clamp hole spacing, with the additional constraint of aligning the fragment's *tassello* within a masonry patch region. The algorithm output in this case includes ten feasible locations on the wall for fr. 421, ranked in order of the fitness of the match (fig. 23).

Obviously, the addition of further constraining features can narrow down the possible wall locations output by the matching process. By including fr. 421 in runs of the boundary incision matching algorithm (described above), we were able to discover a possible match between frs. 421 and 475 (fig. 24). In this hypothetical join, three pairs of incisions align across the slab boundary between the two fragments, and the *ductus* is not dissimilar. Examination of the actual marble fragments in March, 2004 confirmed the plausibility of this match (fig. 25).

Because fr. 475 includes both a clamp hole and a slab corner, its addition to fr. 421 increases the number of wall feature constraints from three to five. When the wall feature matching procedure is executed a second time on the combined frs. 421 and 475, the algorithm outputs only two suggested locations on the wall (fig. 26), as opposed to the original ten, with one of the proposed locations ranked significantly higher in score than the other. This clearly is a more usable result for archaeologists using the computer tools to filter possible fragment locations based on geometric constraints.

The highest probability position for fragments 421 and 475 suggested by the wall feature matching process corresponds to a placement on the Aventine, with fr. 421 along the top center of the empty slab to the right of Circus Maximus frs. 8, and fr. 475 in the bottom right corner of the slab containing frs. 7 (fig. 27). The clamp holes on frs. 475 and frs. 421ab correspond with Cozza's clamp holes B7-3 and B7-5, respectively. There are a number of additional factors which can be considered to support such a positioning. Since fr. 475 would be located in the same slab as frs. 7, it would be expected to share similar characteristics. Although the marble veining direction between fragments 7 and 475 appears reasonably compatible, the thickness of frs. 7e and 475 differ by almost a centimeter. However, the fragments would be positioned approximately 0.80 meters apart, and this variation in thickness is not unprecedented for areas so far apart within the same slab. One compelling similarity between frs. 7e and 475 is the condition of the marble on the bottom slab edge surface; both fragments have a smooth textured band of the same width (ca. 45 mm) along this edge, and also exhibit a very similar curved pattern of sawing marks.

Because the suggested position of fr. 421 is in a previously unoccupied slab, there are no like comparisons that be made to evaluate its suitability here. It is worth noting, however, that the predominate angle of the dense architecture and streets on frs. 421ab is the same (28-30 degrees from vertical) as that on nearby fr. 21, and of excavated structures along the Clivus Publicius. Again, the tentative placement of a fragment into an empty slab determines a number of properties of that slab and invites a number of new hypotheses. In fig. 28 we add fr. 422 based on its strong similarities to fr. 421 (thickness, *ductus*, angle between architecture and veining direction), and align the large street along the Clivus Publicius extending from fr. 21.

Multivariate Clustering

Another automated fragment matching technique that we have developed for aiding reconstruction is multi-variable clustering. The goal of the clustering algorithm is to identify new pairings or groupings of fragments which may not have explicitly corresponding incisions, but may share other common characteristics including fragment thickness, marble veining direction, axial direction of the incised architecture, presence and orientation of slab edges, and the back surface condition of the fragments (rough, smooth, or unpreserved). As with the other matching techniques, the algorithm searches through the database of digitized and annotated fragments, and assigns high scores to those groups of fragments that have a high degree of correlation based on these fragment properties. The clustering procedure can also be used in a more interactive mode, by searching the database for fragments that meet the constraints posed by a user in a specific query. Such a query might be of the conceptual form, "list all of the fragments that have thicknesses in the range 55 - 60 mm and have a veining direction in the range 30 - 45 degrees relative to a slab edge." This capability allows researchers to quickly focus on a suitable subset of fragments when experimenting with possible reconstructions.

Fragment 307

As an example of applying multi-variable clustering, consider the fragments 37f-i, 37Am, and 40 placed by Rodríguez-Almeida in the southwest area of the *campus Martius*[14] (fig. 28). These fragments are grouped together near the bottom right corner of the slab containing frs. 37. The thicknesses of the fragments in this slab form a gradient with values increasing towards the bottom of the slab (from 51 mm on the top of 37g to 59mm on the bottom of 37i), and increasing slightly from right to left. The veining direction angle is 35 degrees from the lower slab edge, and the primary architectural axis on fragments 40ac, 37i and 37Am is approximately 5 degrees from horizontal. With these constraints in mind, we can create a query to find previously unidentified fragments that are similar to this group and may lie within the fr. 37 slab: "list the fragments that have a smooth back surface, thicknesses in the range 55 - 60 mm, veining direction near 35 degrees relative to a slab edge, and a primary architectural axis near 5 degrees relative to a slab edge."

The unidentified fragment in the database that scored highest in response to this query was fr. 307. Fragment 307 has a thickness ranging from 57.5 at one end to 61 mm along its slab edge, and has veining direction and architectural axis angles that correspond very well with those requested. The gradient of thickness within fr. 307 suggests a possible orientation along the lower slab edge near the other fragments in slab 37; one such possible

[14] RODRÍGUEZ-ALMEIDA 1981, pp. 130-139, 149-150, fig. 41.

position is shown in fig. 35. Because the fragment does not contain any vertical text, we position the fragment just to the left of the expected inscription *VICUS STABLARIUS*. The style of the incisions and the architecture on the fragment is reasonably consistent with that on frs. 40 and 37f-i.

Although the clustering and positioning of fr. 307 in the slab with frs. 37 is appropriate based on the criteria employed by the algorithm, it is not enough evidence to adequately confirm such a location. This is typical of results generated by the multi-variable clustering approach; it is useful for rapidly classifying fragments into plausible slab groupings (particularly for fragments with slab edges), but does not find fragment joins of immediately high certainty as can approaches that search specifically for directly corresponding, complementary geometric features.

Edge Fracture Geometry Matching

A final computer-aided reconstruction approach that we are experimenting with is edge fracture geometry matching. This family of techniques attempts to use our three-dimensional scanned data of the fragments as input, and search for matching shapes among all of their fractured edges. In one approach, we extract two-dimensional boundary slices from the three-dimensional models at different levels of the fragment thickness (fig. 29), and then convert these two-dimensional contours to one-dimensional signals by reparameterizing the curves along their arc length. These one-dimensional representations of the fractured edge surfaces can be searched and compared for match quality very efficiently, using well-studied matching algorithms developed by computer scientists for applications such as searching large bodies of text and sequencing the human genome. Although we initially expected this shape matching of scanned fragment edges to be the most fruitful approach for digital reconstruction of the Plan, the large degree of erosion of many of the fragment edges may be a limiting factor in the usefulness of these techniques.

Conclusion

Although we are still fine-tuning and experimenting with our digital reconstruction algorithms, several of the methods have been successful for suggesting new fragment matches. Because encoding all archaeological information is probably impossible, we use the computer primarily for searching and evaluating geometric constraints. Human expertise is still required to prepare annotated input data, verify suggested matches, identify false positives, and take into account those factors that are not considered in the automated process.

Systematic computer searching for fragment matches highlights the importance of dealing with uncertainty in proposed fragment joins and placements. The fitness scores output by the computer algorithms are one attempt to assess proposals in a probabilistic manner, and further analysis under human expertise serves to refine and rank these scores. In some cases, new fragment matches suggested by the computer can be confirmed with an extremely high degree of certainty, such as those involving frs. fn23/28a/34b, fn9/351, 28a/150, 92/138, and 330/354 described above. More generally, however, the digital analysis process has resulted in hundreds of proposals of fragment positionings with widely ranging levels of uncertainty. We are currently developing tools and techniques for representing and visualizing this uncertainty, and we continue to scrutinize the output of our existing digital reconstruction algorithms for new, enlightening proposed fragment matches. Further updated information on these results will be available at the Stanford Digital Forma Urbis Project website[15].

[15] The Project website address is *http://formaurbis.stanford.edu*.

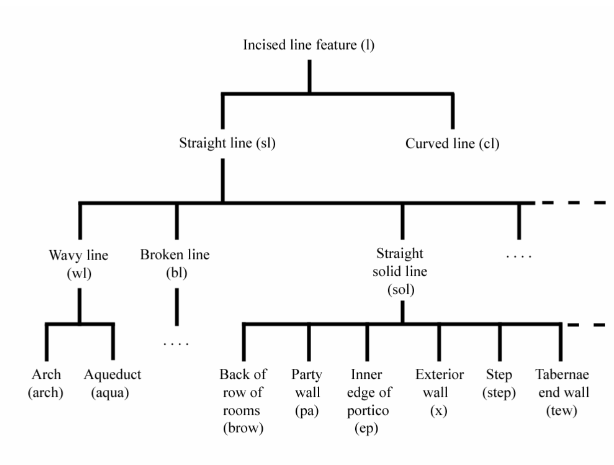
Abbreviations

RODRÍGUEZ-ALMEIDA 1981 = E. RODRÍGUEZ-ALMEIDA, *Forma Urbis Marmorea: Aggiornamento generale 1980*, Roma 1981.

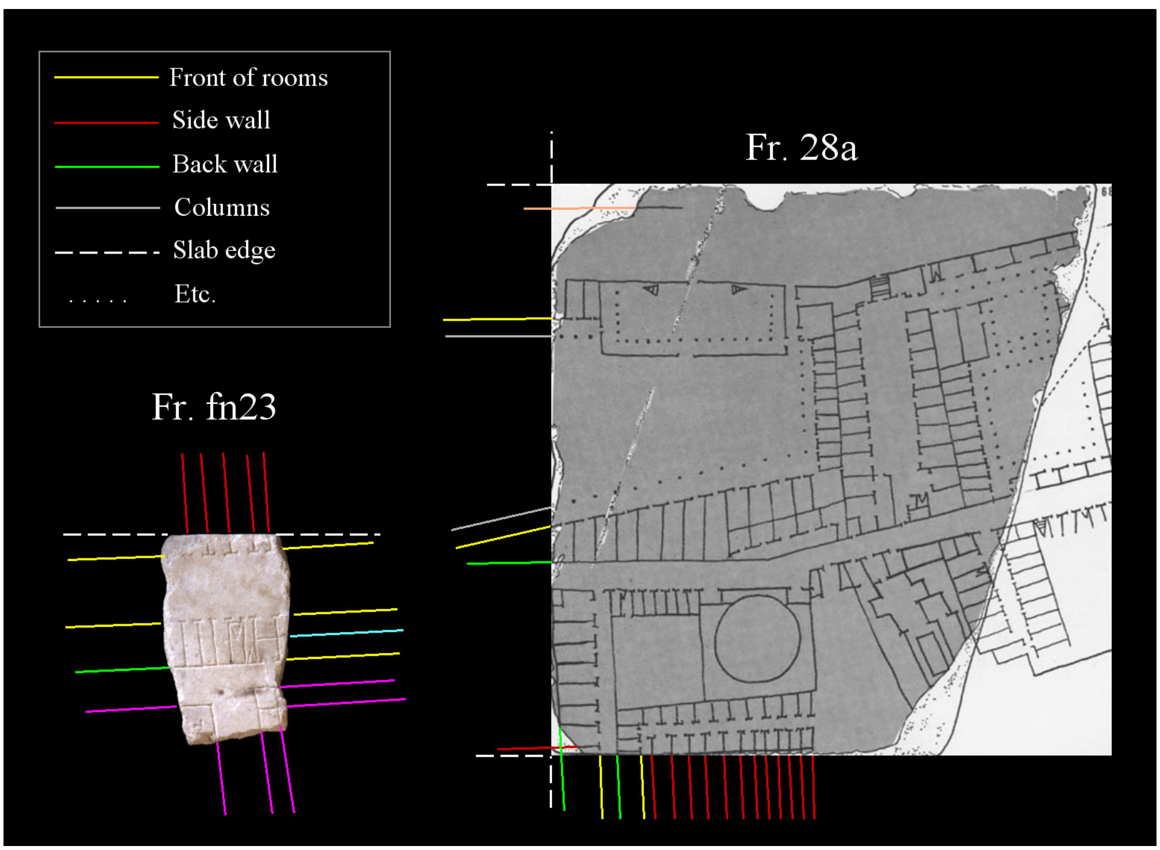
RODRÍGUEZ-ALMEIDA 1992 = E. RODRÍGUEZ-ALMEIDA, *Novita minori dalla Forma Urbis marmorea*, in *Ostraka*, 1, 1992, pp. 55-80.

CARETTONI-COLINI-COZZA-GATTI 1960 = G. CARETTONI, A.M. COLINI, L. COZZA, E. GATTI, *La pianta marmorea di Roma antica*, Roma 1960.

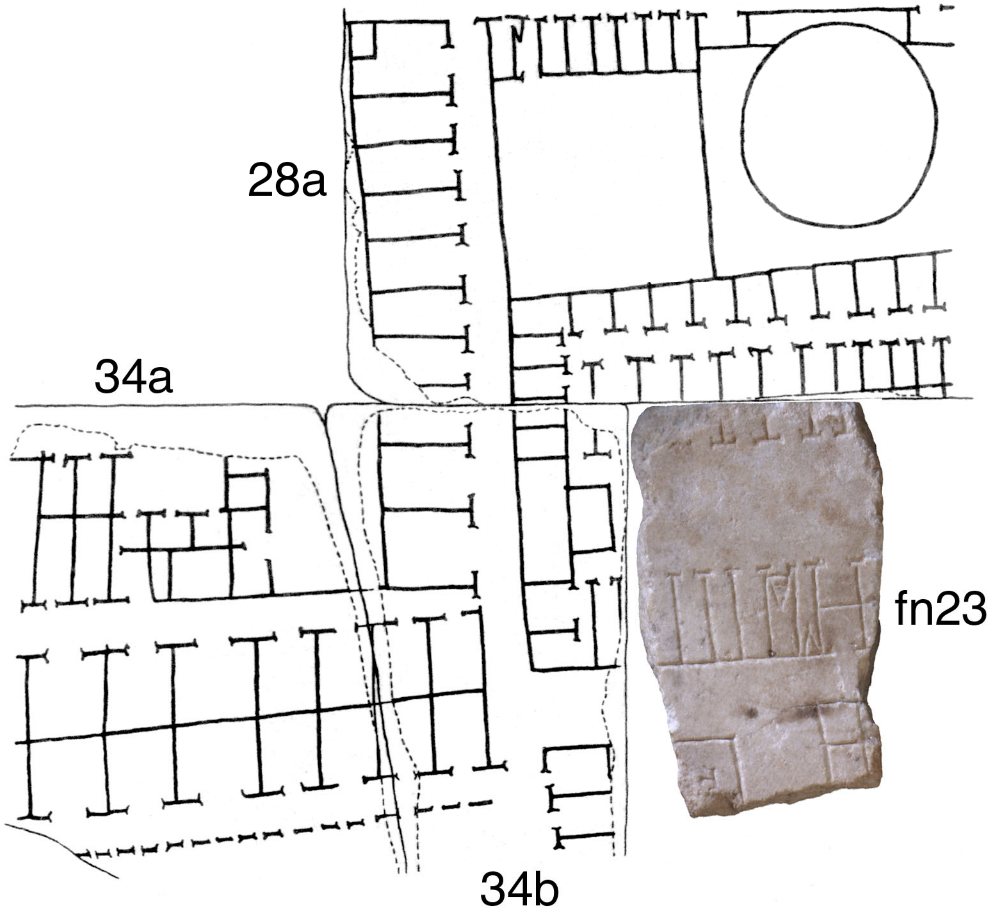
Figures for *Computer-aided Reconstruction and New Matches in the Forma Urbis Romae*, by David Koller and Marc Levoy



1. A small portion of the hierarchy of incised feature types used to label the boundary incisions.

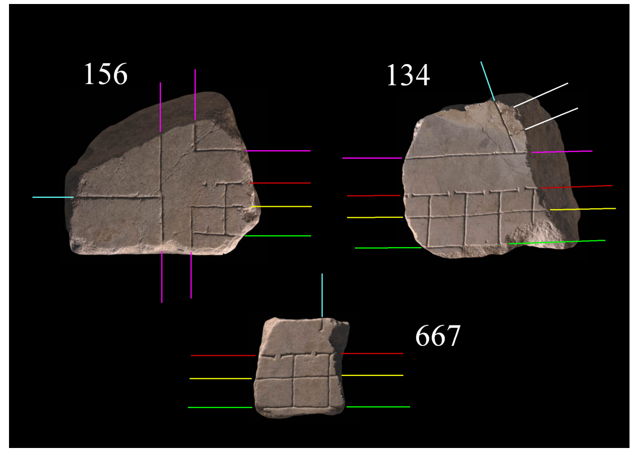


2. Fragments fn23 and 28a with their boundary incision annotations indicated. The different colors correspond to different feature type labels (fr. 28a illustration from RODRÍGUEZ-ALMEIDA 1981).



3. Fragment fn23 positioned in the corner between fragments 28a and 34b (adapted from CARETTONI-COLINI-COZZA-GATTI 1960).

4. Verifying the alignment of the incisions on fragments fn23 and 34b.

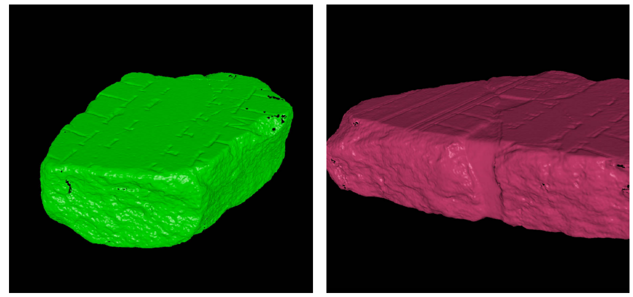


5. Fragments 156, 667 and 134 with their boundary incision annotations indicated.

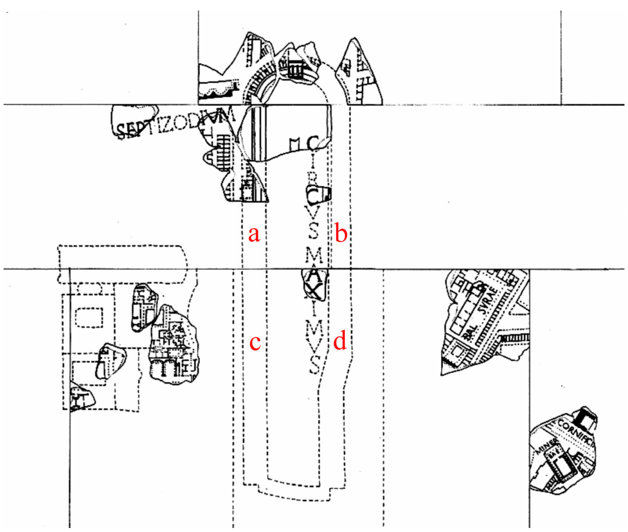
6. Digital composite image of a proposed position for fragment 156 adjacent to fragments 667/134.

7. Fragments 134, 667, and 156.

8. Digital composite image of fragments fn9 and 351.



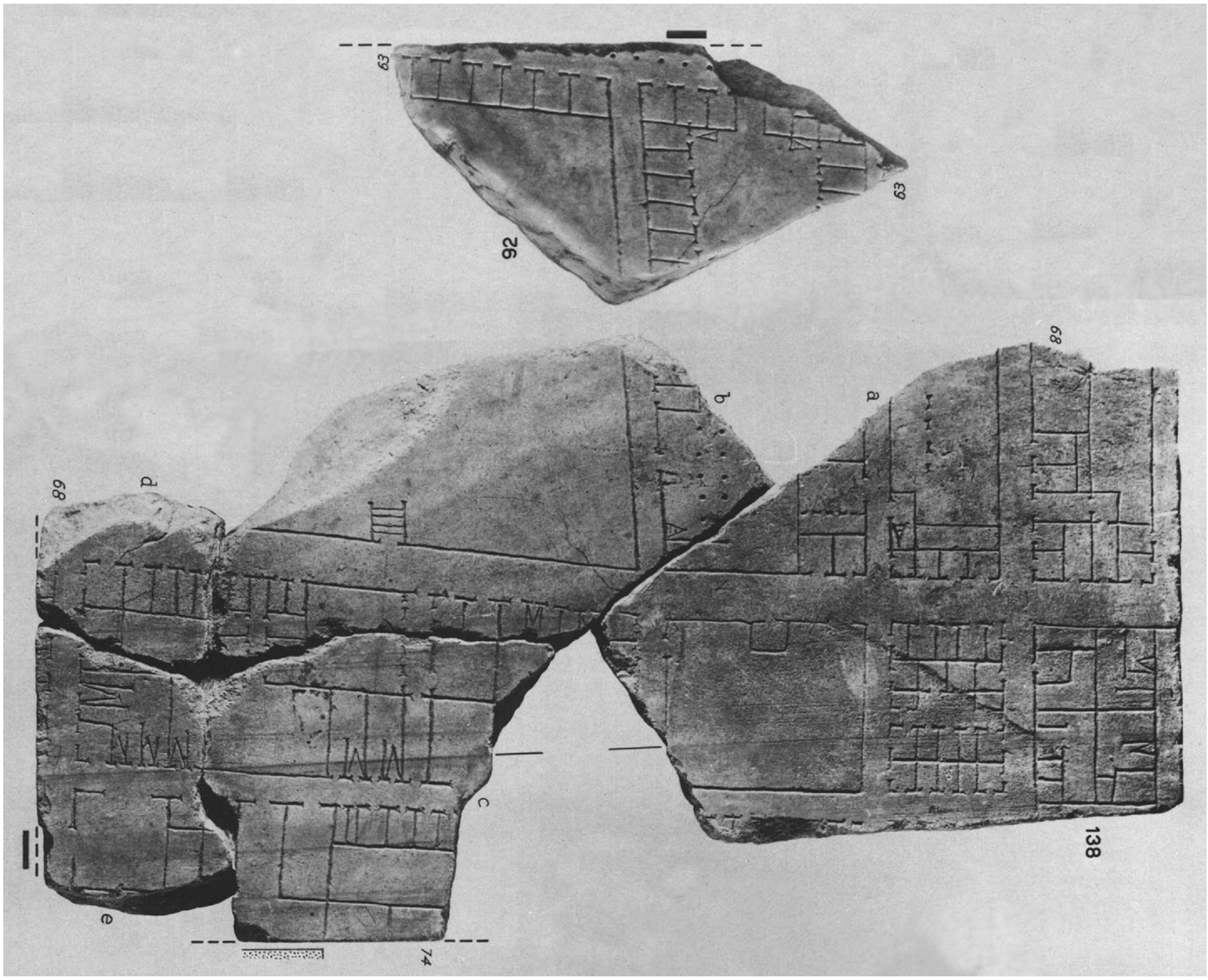
9. Corresponding fractured edge shapes visible in the computer models of frs. 351 (left) and fn9 (right). The generally concave form of fr. 351 mates with the convex edge of fn9.



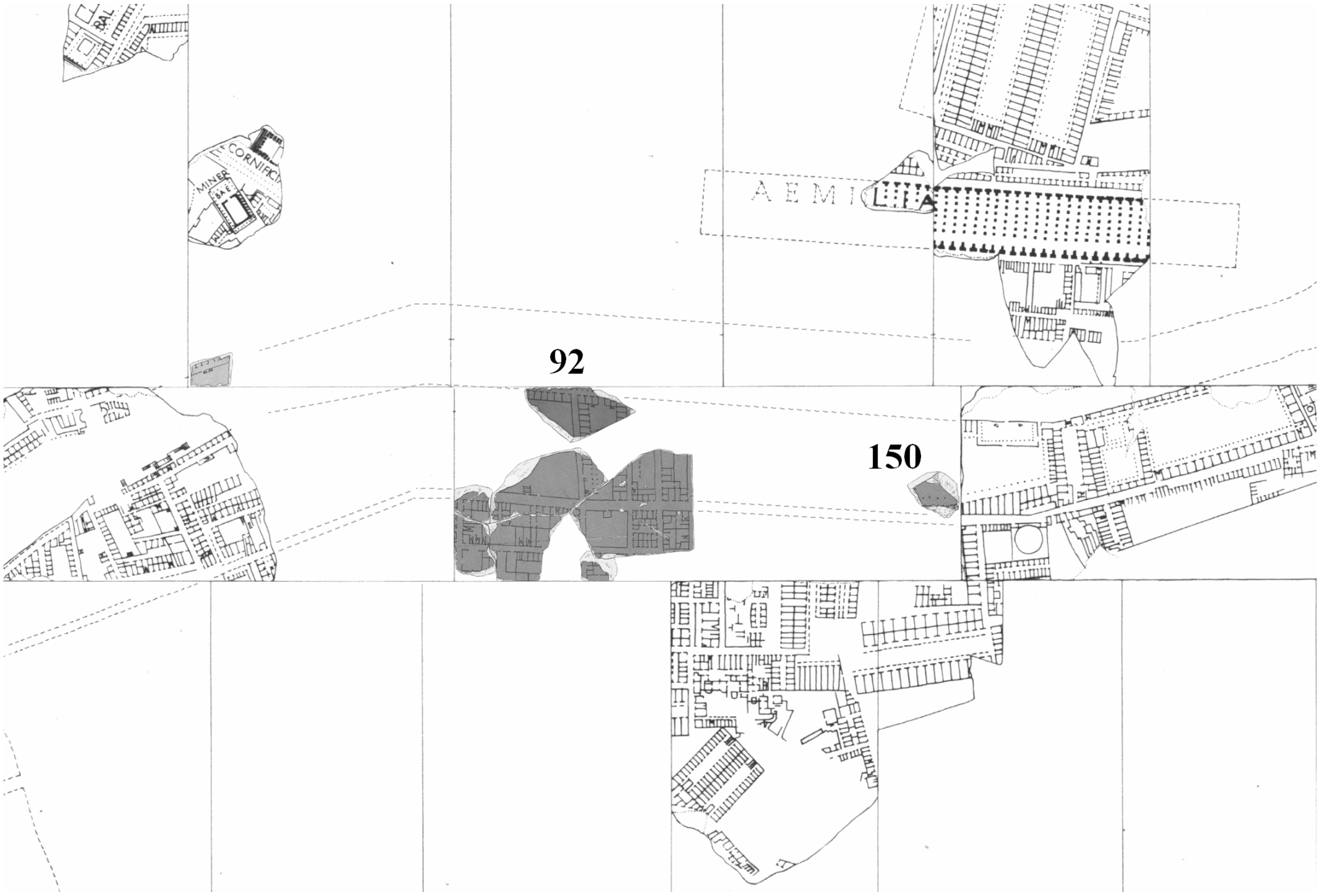
10. Possible positions (a-d) for frs. fn9 and 351 along the sides of the Circus Maximus (from CARETTONI-COLINI-COZZA-GATTI 1960).

11. Digital composite image with frs. fn9/351 positioned along the lower left side of the Circus Maximus.

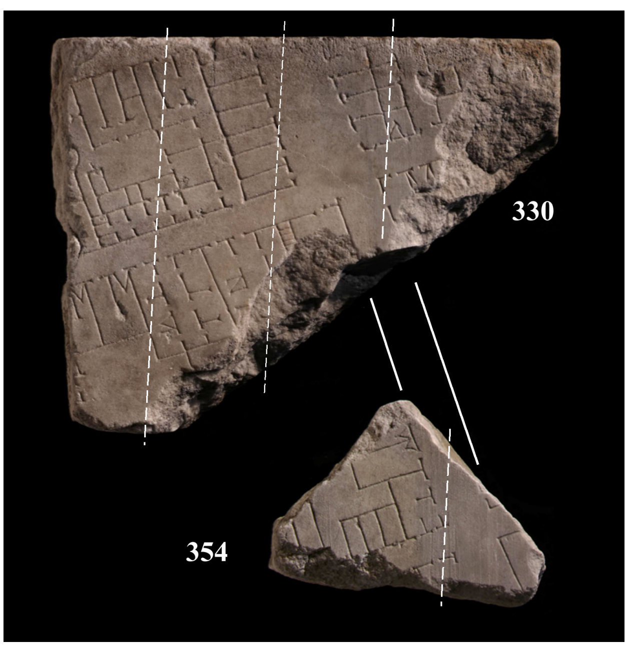
12. Composite image of fragments 150 and 28a.



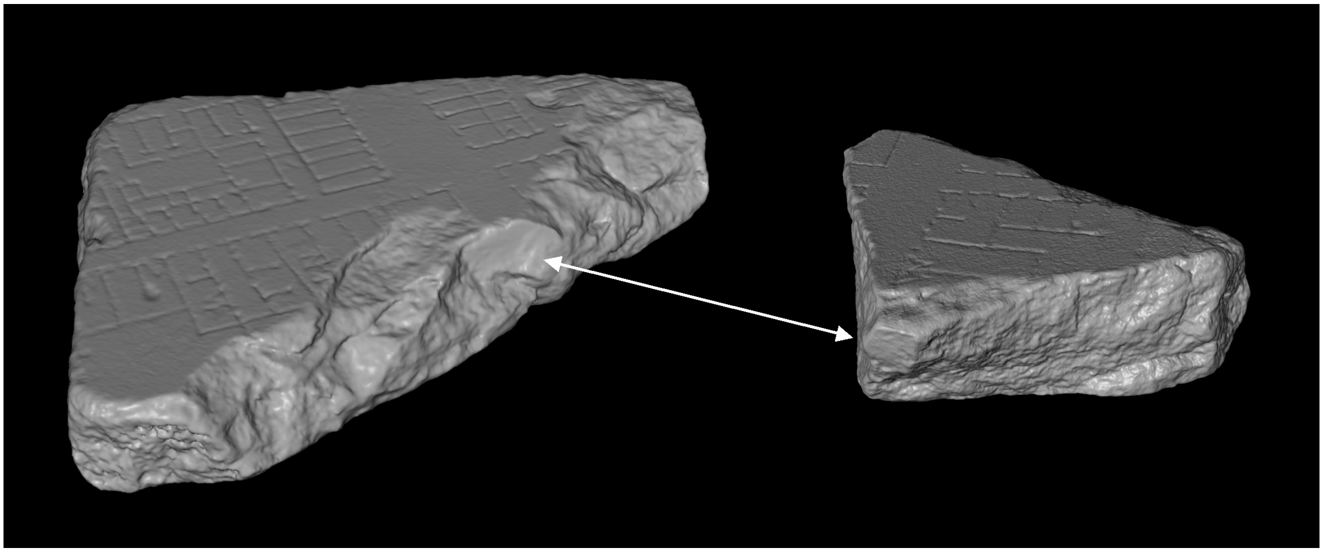
13. Composite image of fragments 92 and 138abcde in the proposed relative position (composited from CARETTONI-COLINI-COZZA-GATTI 1960).



14. Fragments 92 and 150 positioned on the right bank of the Tiber (adapted and composited from CARETTONI-COLINI-COZZA-GATTI 1960 and RODRÍGUEZ-ALMEIDA 1981).

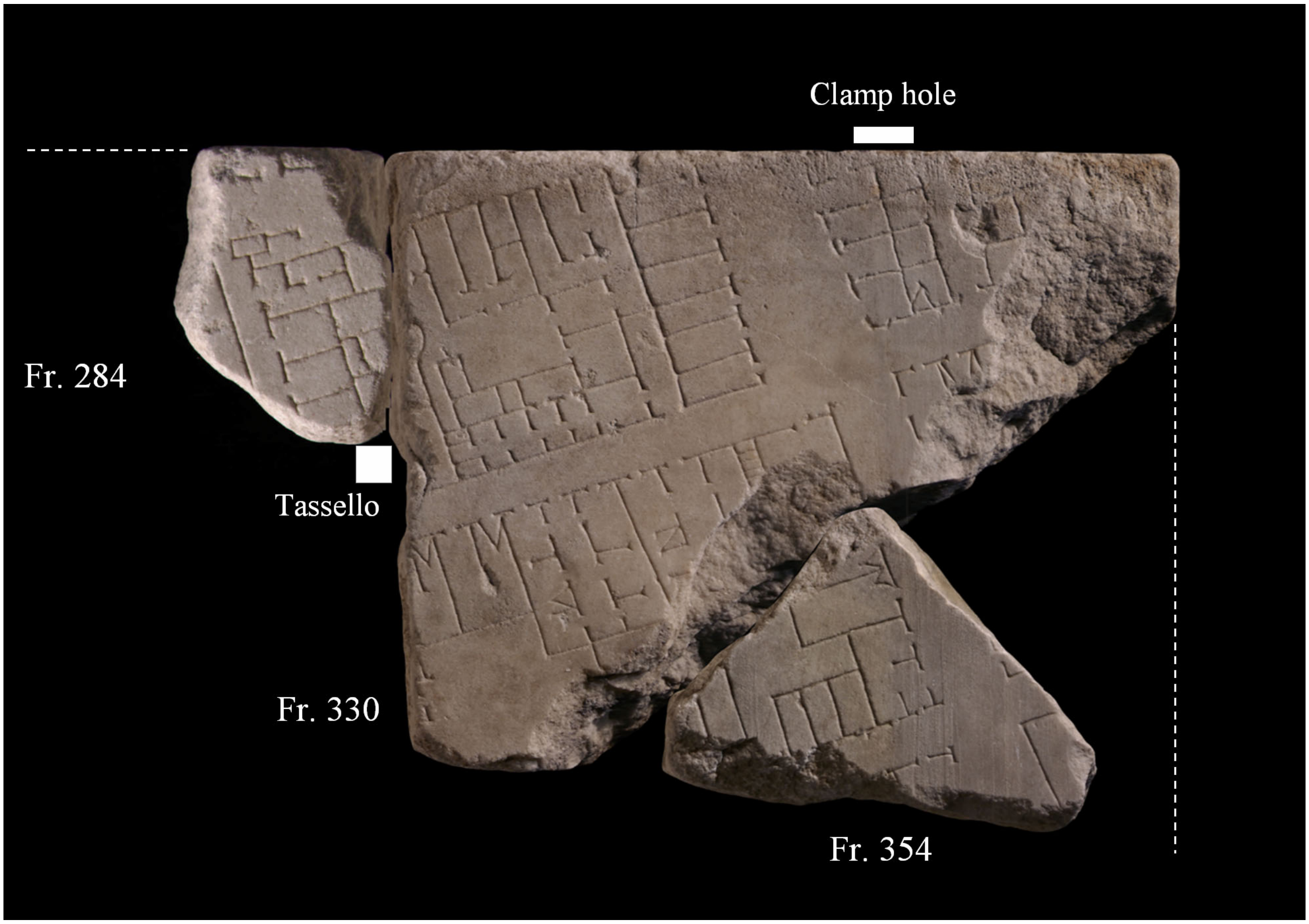


15. Matching incisions (solid) and veining direction (dashed) constraints between frs. 330 and 354.

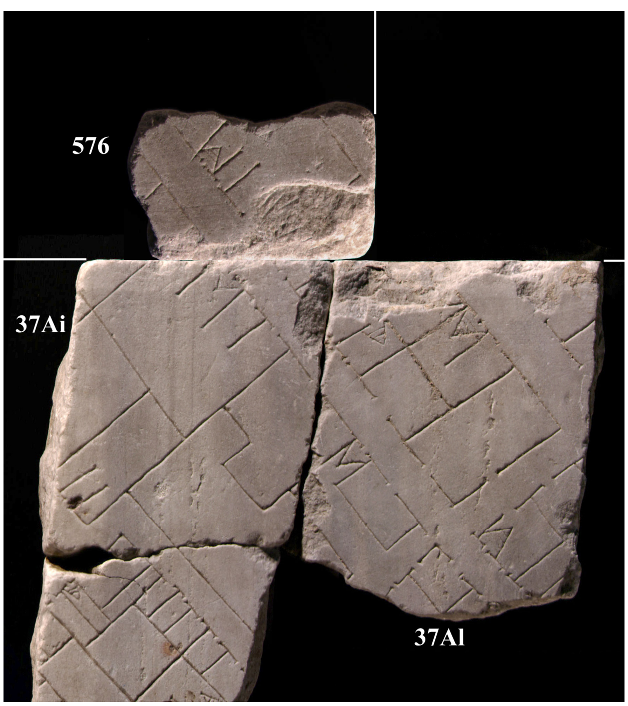


16. Three-dimensional computer models of fragments 330 (left) and 354 (right), showing the unusual smooth, flat sections along the fractured edge surfaces near the proposed join.

17. Marble fragments 330 (left) and 354 (right) joined together.

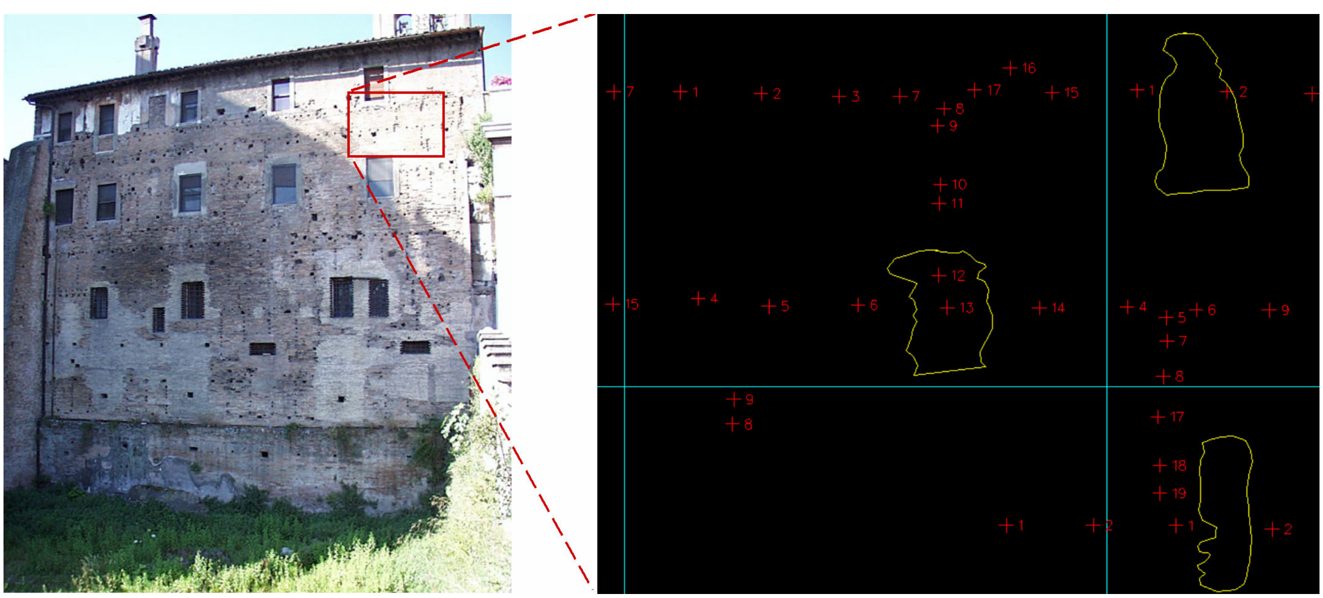


18. Composite image of fragments 284, 330, and 354.

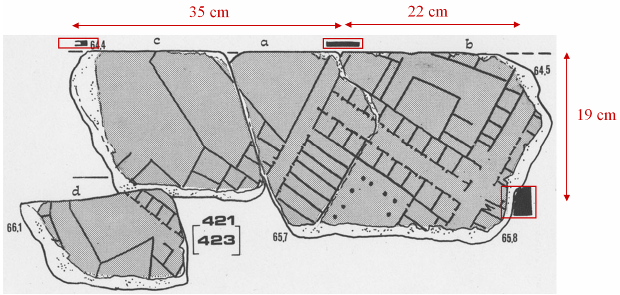


19. Composite image of fragment 576 in a suggested position above frs. 37Ail.

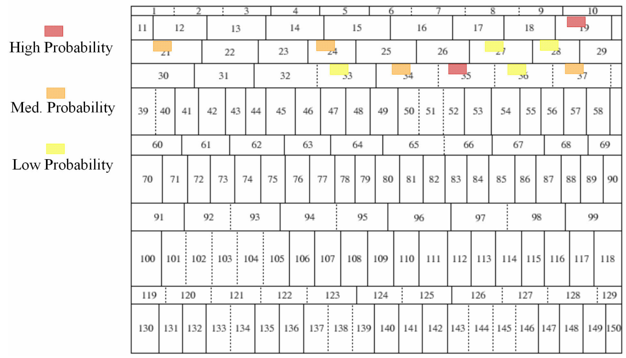
20. Composite image of fragments 141 and 194 in a suggested relative position.



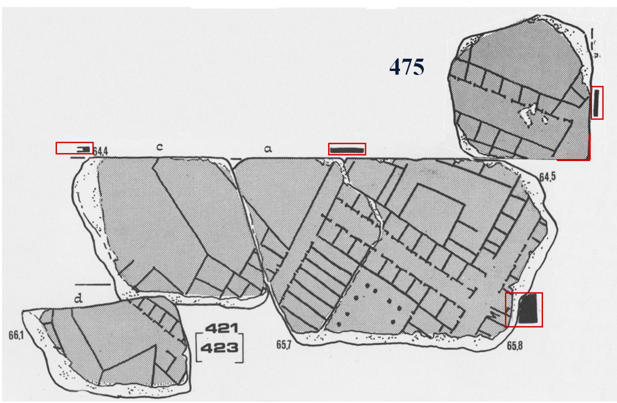
21. The aula wall (left) and a section of the digitized wall features (right), with clamp hole locations indicated in red and masonry patches in yellow.



22. Fragment 421, exhibiting two clamp holes and one *tassello* constraint (adapted from RODRÍGUEZ-ALMEIDA 1981).

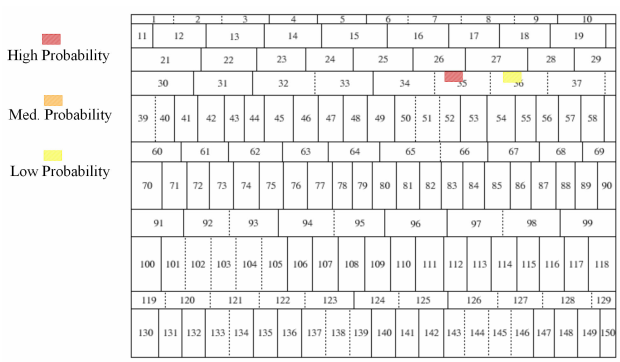


23. Wall feature matching algorithm results suggesting fr. 421 positions, using three constraining features.

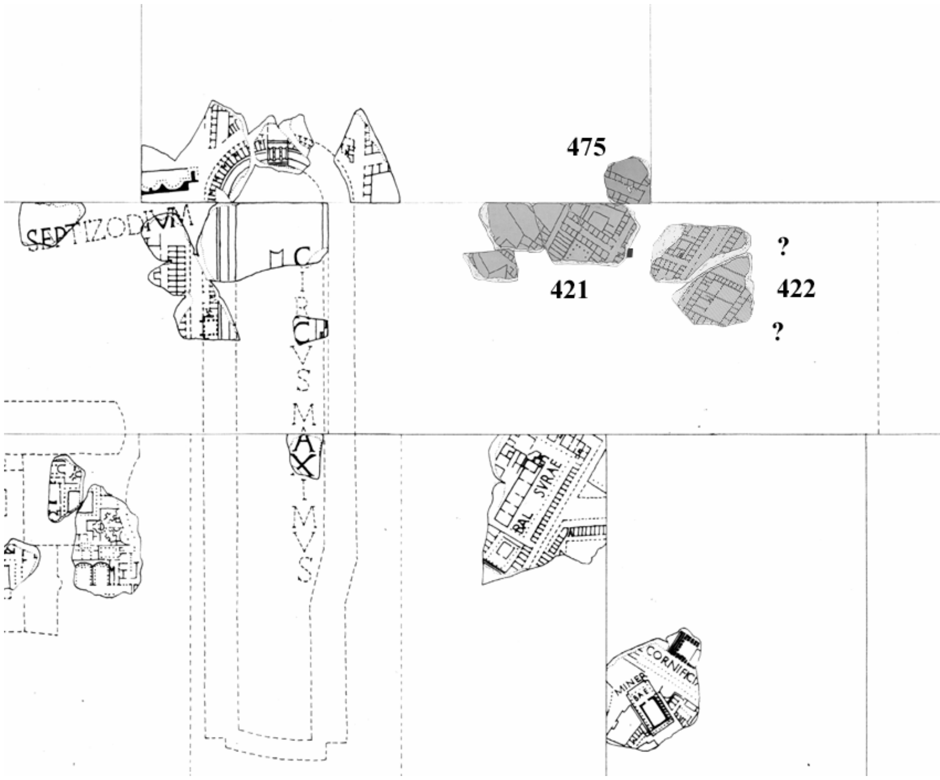


24. Proposed match between fragments 421 and 475, exhibiting three clamp holes, one *tassello*, and one slab corner constraint (composited from RODRÍGUEZ-ALMEIDA 1981).

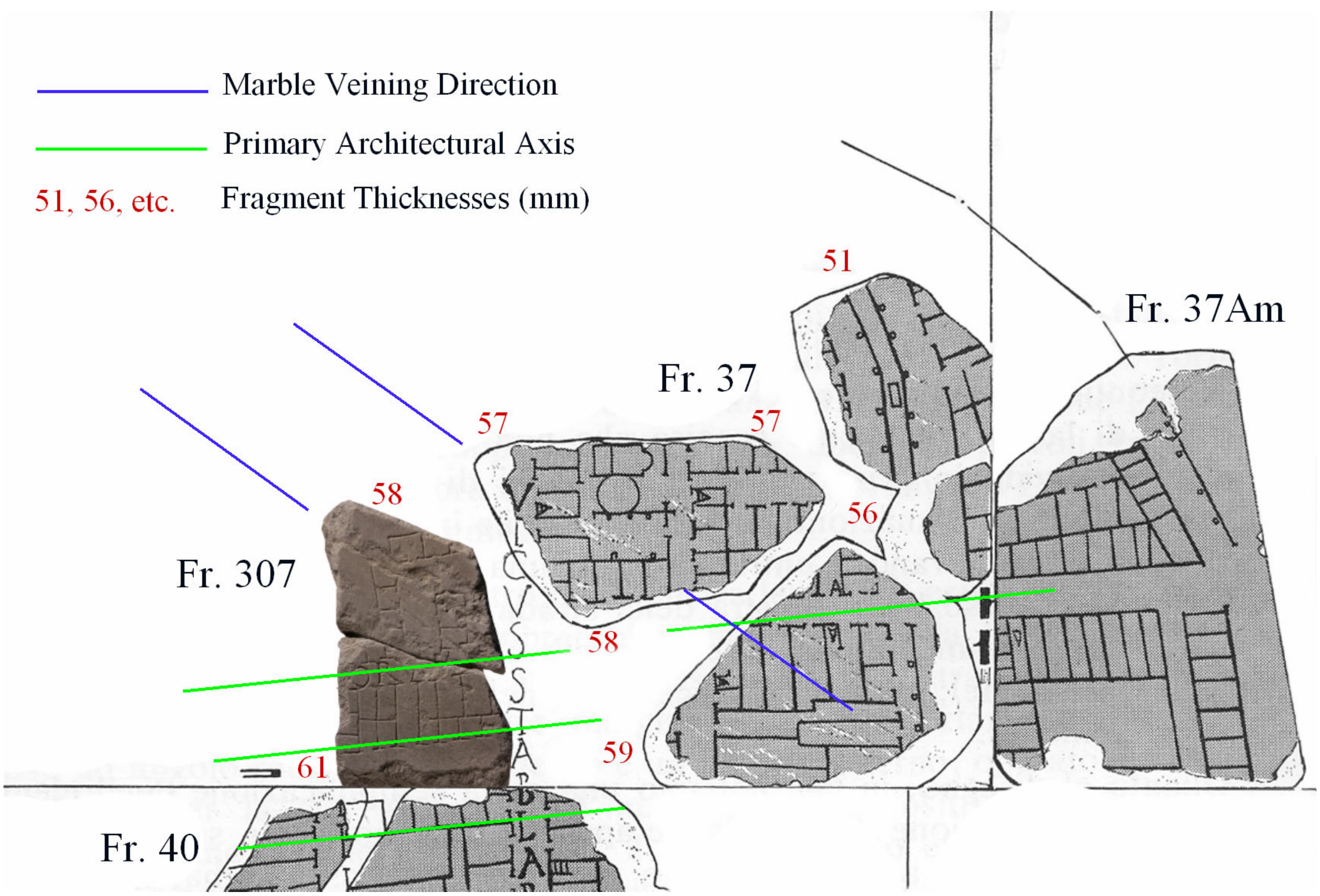
25. Detail of the interface between fragments 421b (bottom) and 475 (top) in the proposed position.



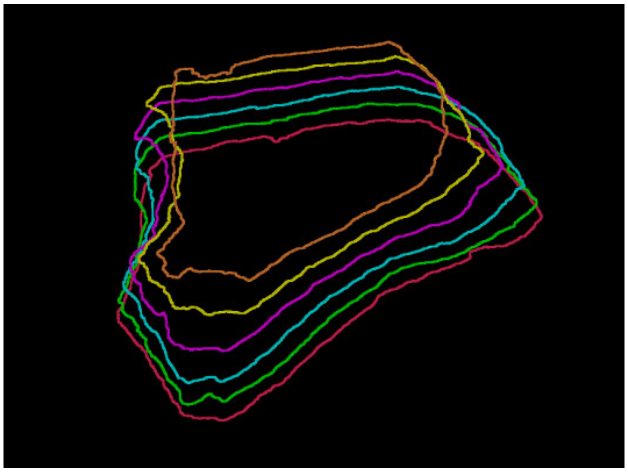
26. Wall feature matching algorithm results suggesting combined frs. 421/475 positions, using five constraining features.



27. Proposed locations for fragments 421, 422, and 475 on the Aventine (composited from CARETTONI-COLINI-COZZA-GATTI 1960 and RODRÍGUEZ-ALMEIDA 1981).



28. Fragment 307 in a candidate position determined by multi-variable clustering with fragments 37fghi, 37Am, and 40ac (composite image adapted from RODRÍGUEZ-ALMEIDA 1981).



29. Two-dimensional boundary curves extracted from a three-dimensional fragment model at 1.0 cm intervals.