Differential Geometry for Computer Science

CS 468, Spring 2013
Differential Geometry for Computer Science

Justin Solomon and Adrian Butscher

<administrative>

Course Instructors

Instructor: Justin Solomon (me)
Email: justin.solomon@stanford.edu
Office: Clark S297
Office hours: After lecture

Instructor: Adrian Butscher
Office: Clark S257
Email: butscher@stanford.edu
Office hours: After lecture

On the Web

`http://cs468.stanford.edu`

+ piazza

Register ASAP!

Expectations

1. **Four homeworks**
   Written + coding

2. **One project**
   Implementation

3. **One set of notes**
   Due one week after lecture

Expectations

1. **Four homeworks**
   Written + coding   *Challenging!*

2. **One project**
   Implementation

3. **One set of notes**
   Due one week after lecture

Lecture Pairs

- **Theory lecture**
  Introduction to concepts from continuous differential geometry

- **Discrete lecture**
  Algorithms and constructions for use in computational systems

Quick Survey

Degree
Undergraduate
Masters
PhD

Quick Survey

Background

CS
Math
Engineering
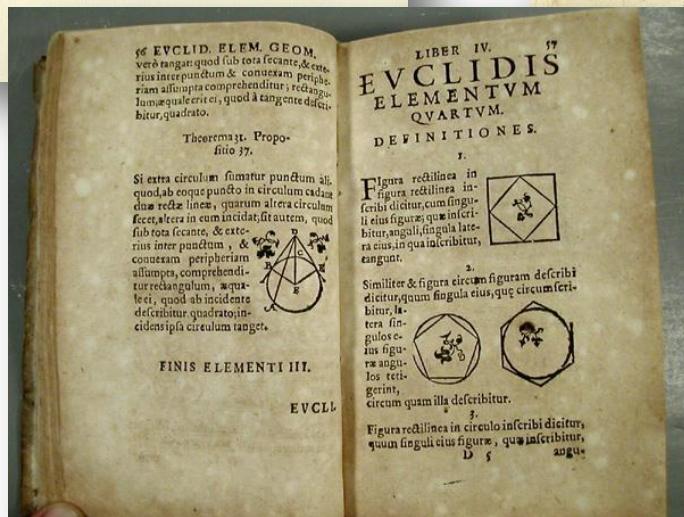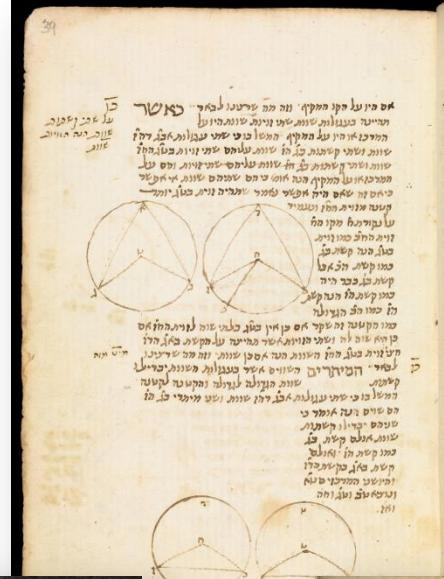## Elsewhere
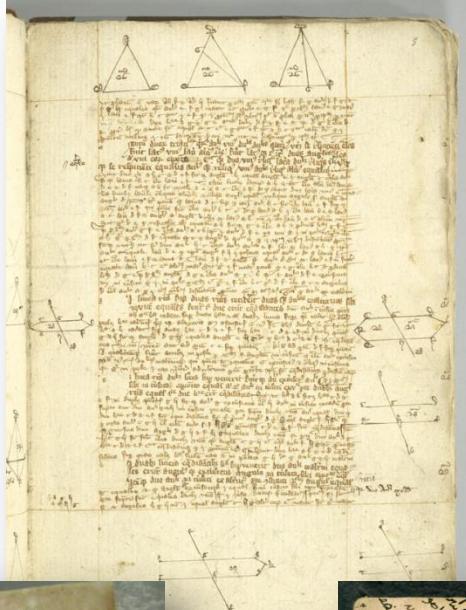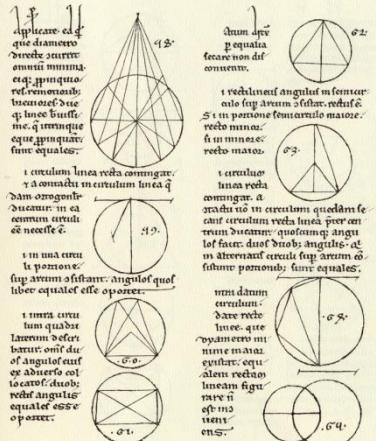
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Two Parallel Threads
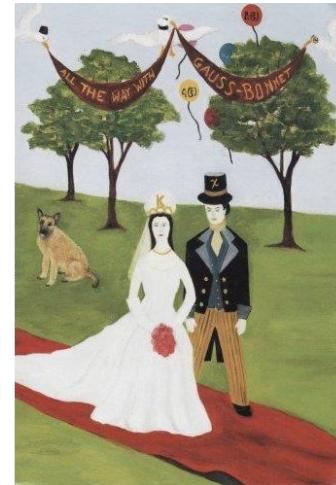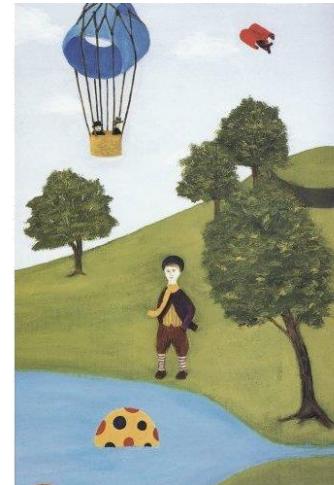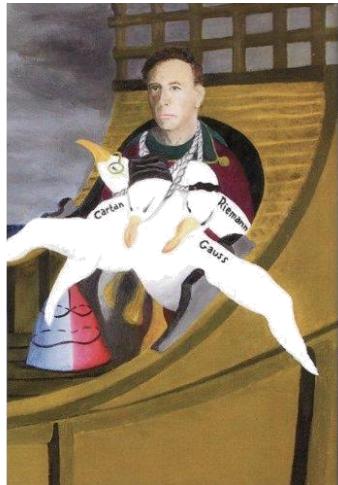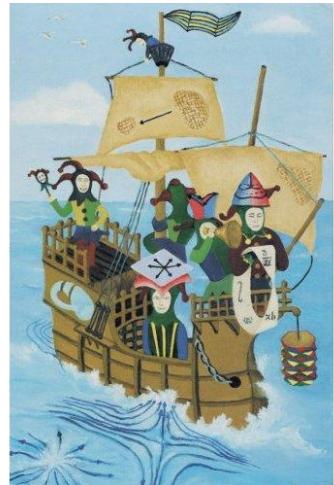
Spivak: *A Comprehensive Introduction to Differential Geometry*

1. *Continuous* differential geometry

Euclidean Geometry
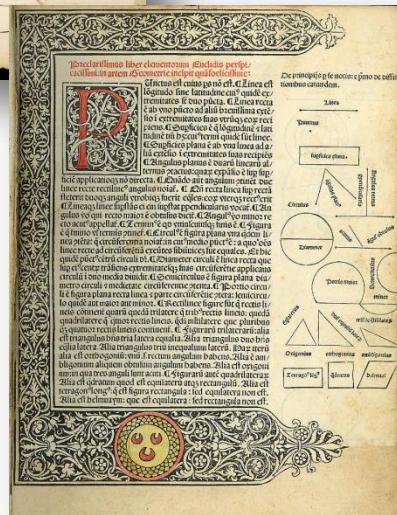
Euclidean Geometry
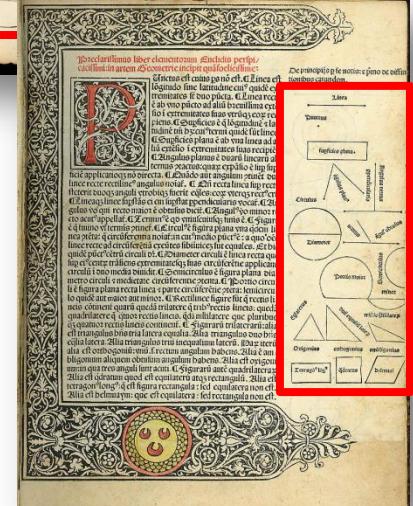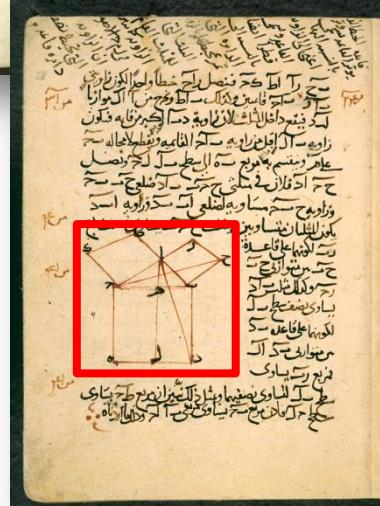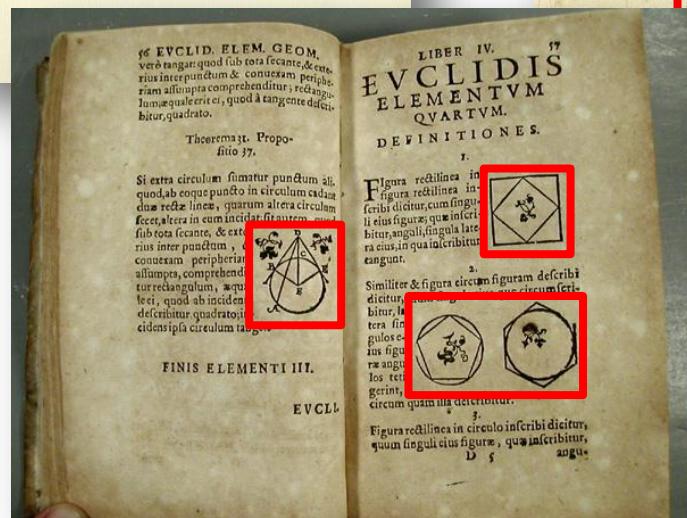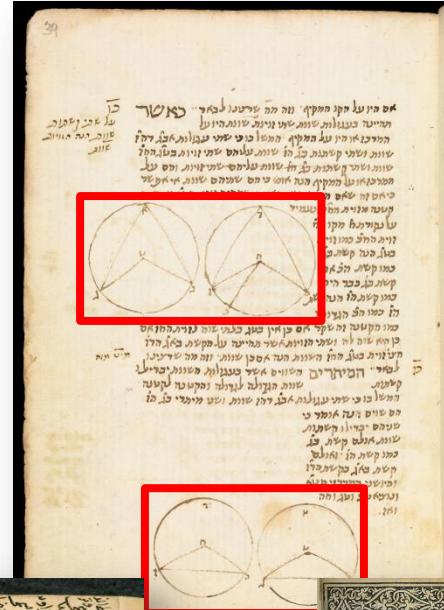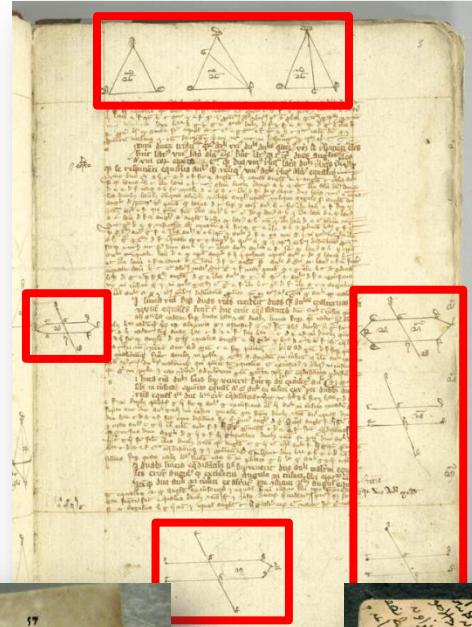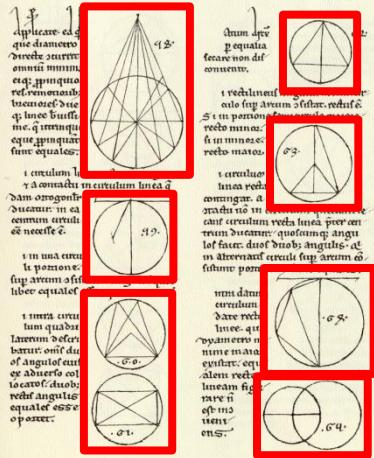
Euclidean Geometry
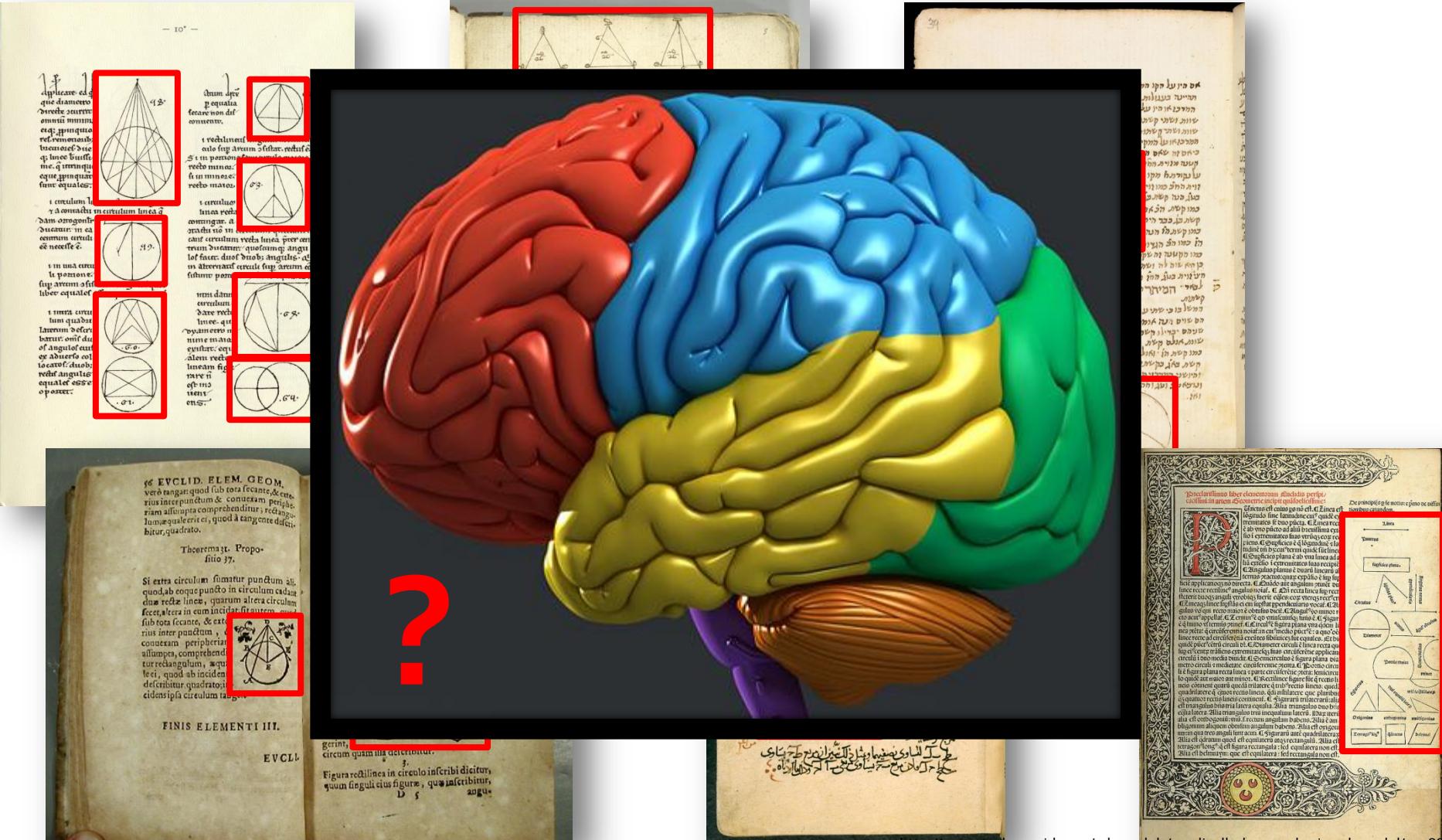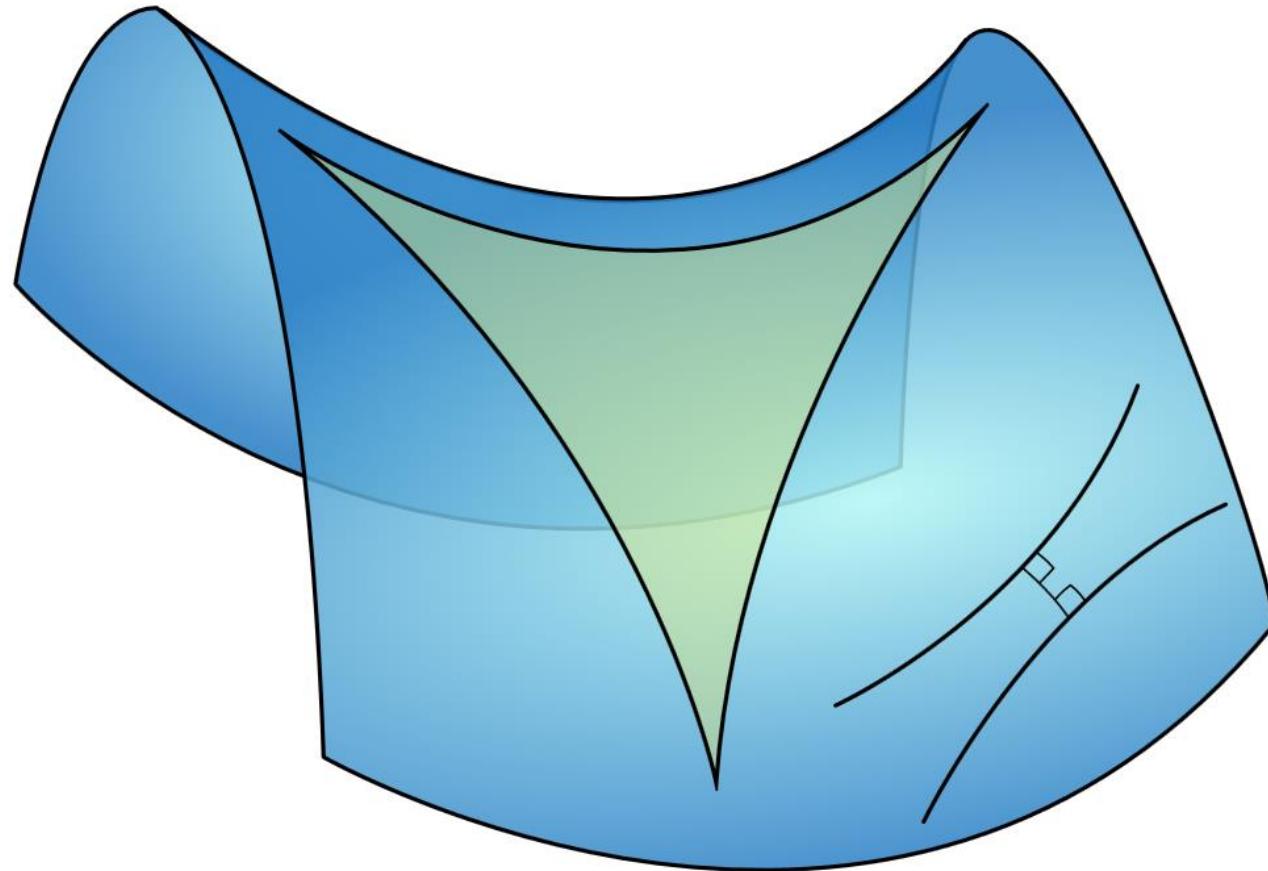
Differential Geometry

Study of smooth surfaces
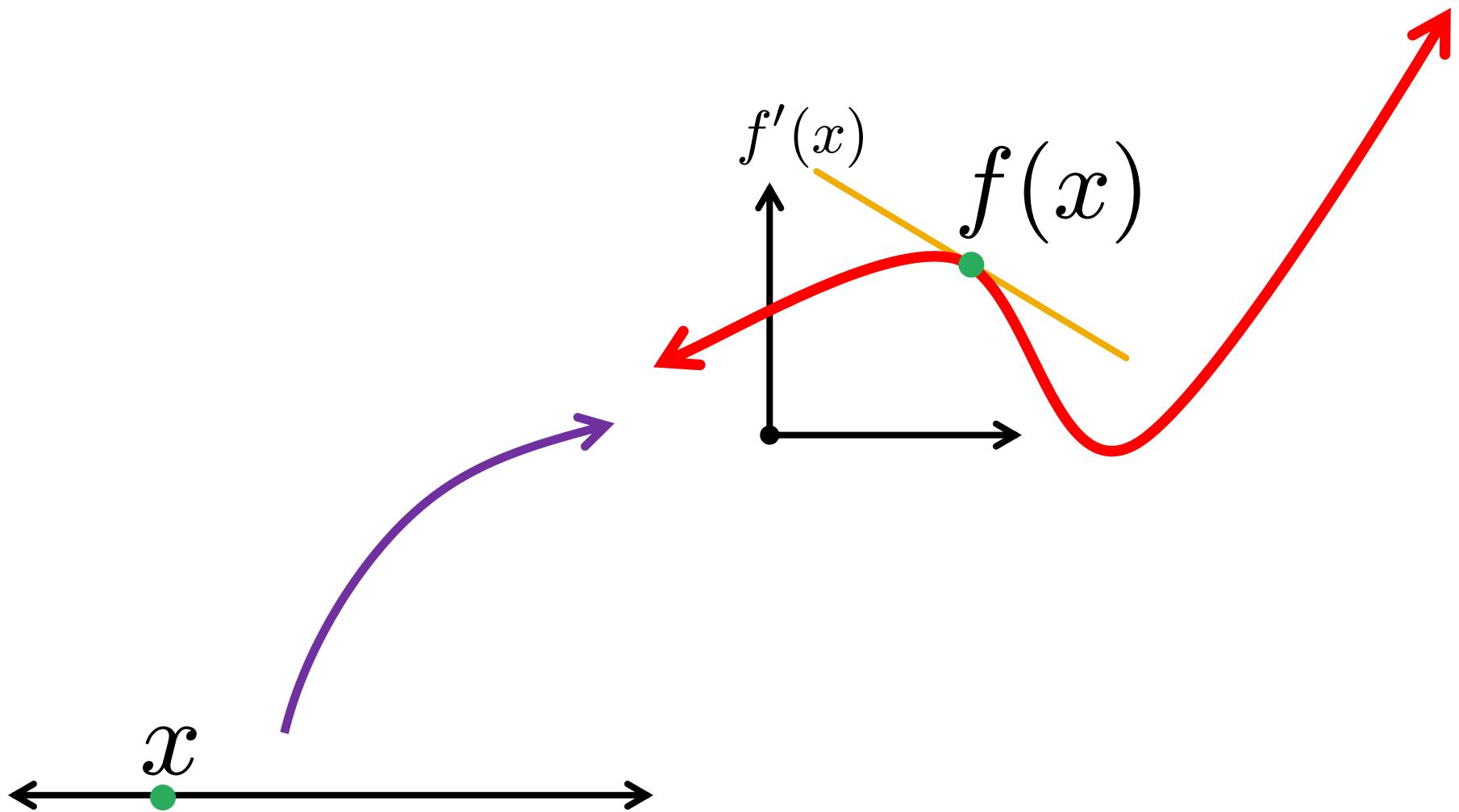
What is a smooth surface?

Not the Best Starting Point
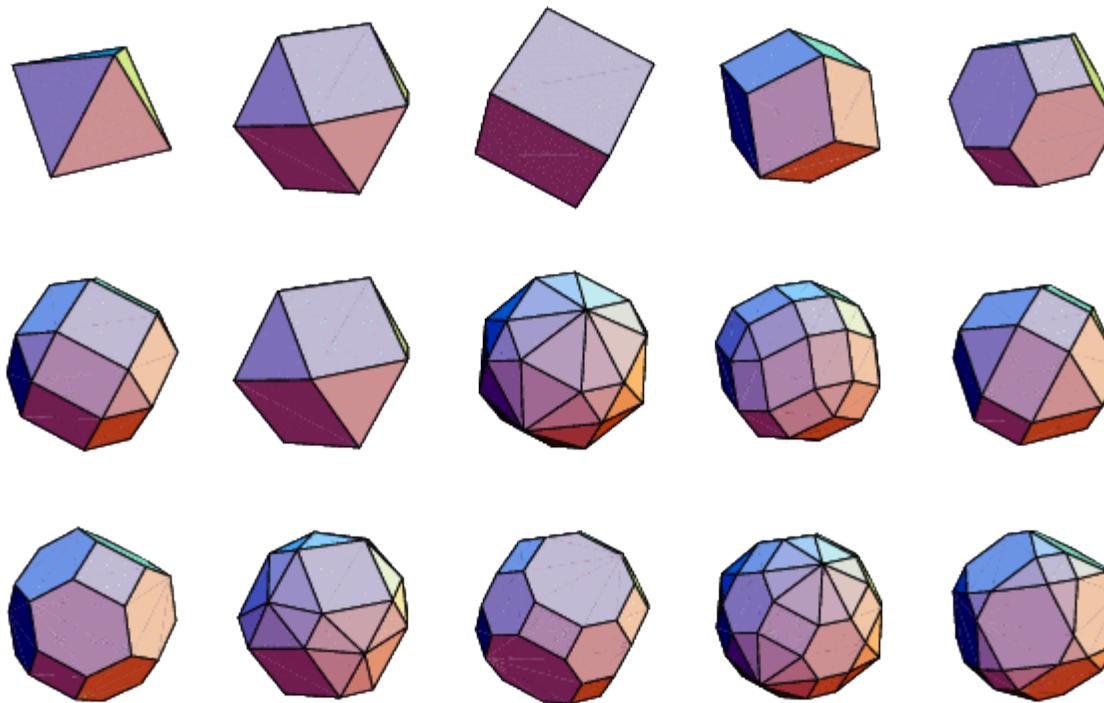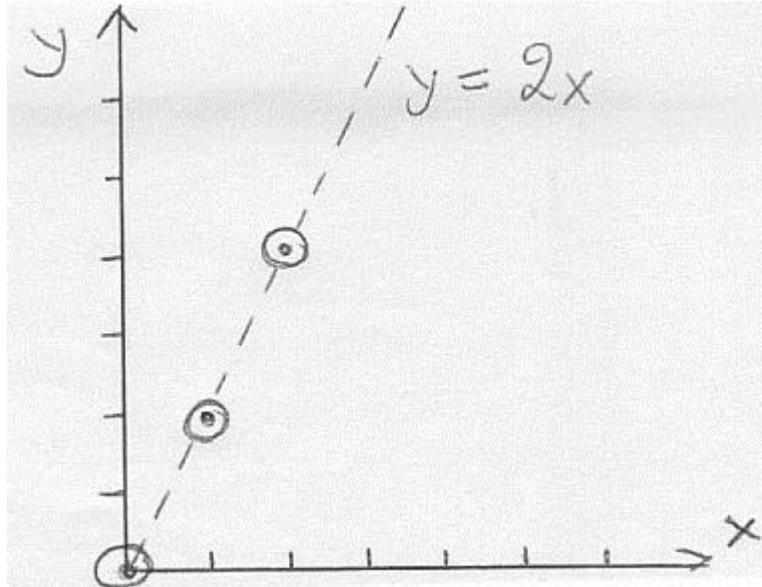
Smooth Function

C^∞

Arbitrary **derivatives** exist and are continuous.
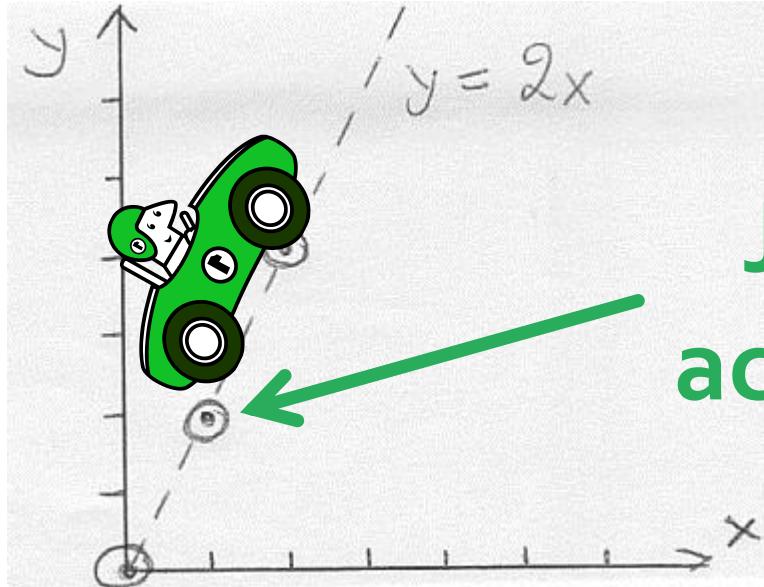
Divergence from Calculus

$$f_1(t) = (t, 2t)$$

$$f_2(t) = \begin{cases} (t, 2t) & t \cdot\ 1 \\ (2(t - \frac{1}{2}), 4(t - \frac{1}{2})) & t > 1 \end{cases}$$

Divergence from Calculus
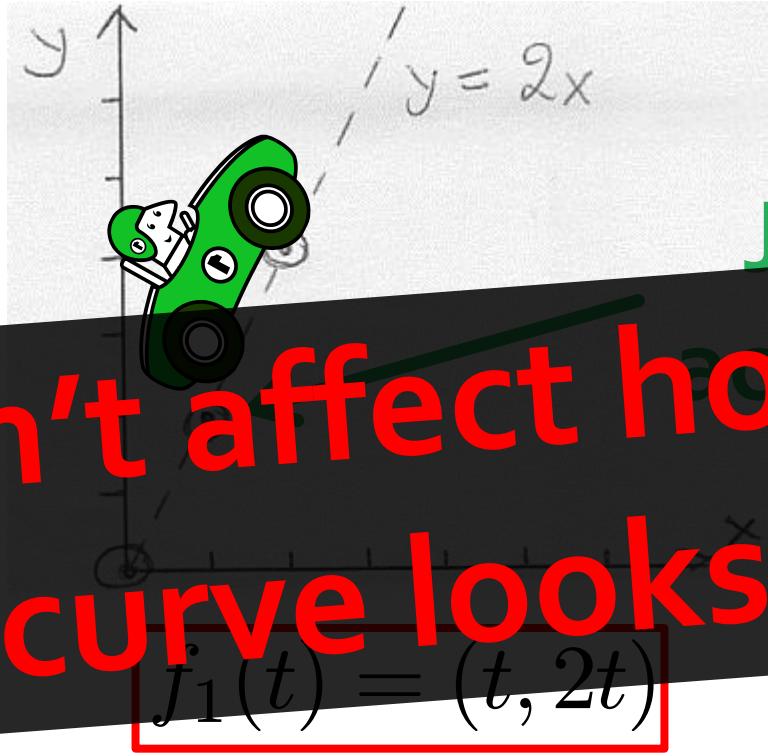

Jams on accelerator

$$f_1(t) = (t, 2t)$$

$$f_2(t) = \begin{cases} (t, 2t) & t \cdot 1 \\ (2(t - \frac{1}{2}), 4(t - \frac{1}{2})) & t > 1 \end{cases}$$

Divergence from Calculus
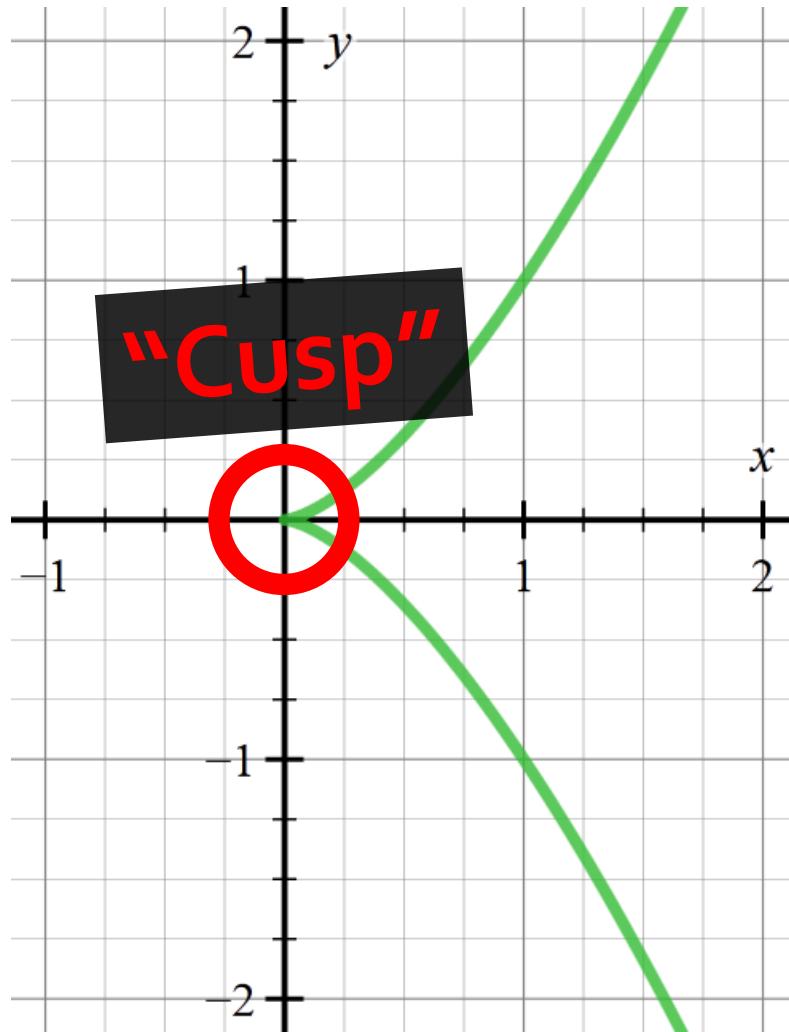

Jams on ~~acceleration~~

Doesn't affect how the curve looks.

$$f_1(t) = (t, 2t)$$

$$f_2(t) = \begin{cases} (t, 2t) & t \leq 1 \\ (2(t - \tfrac{1}{2}), 4(t - \tfrac{1}{2})) & t > 1 \end{cases}$$
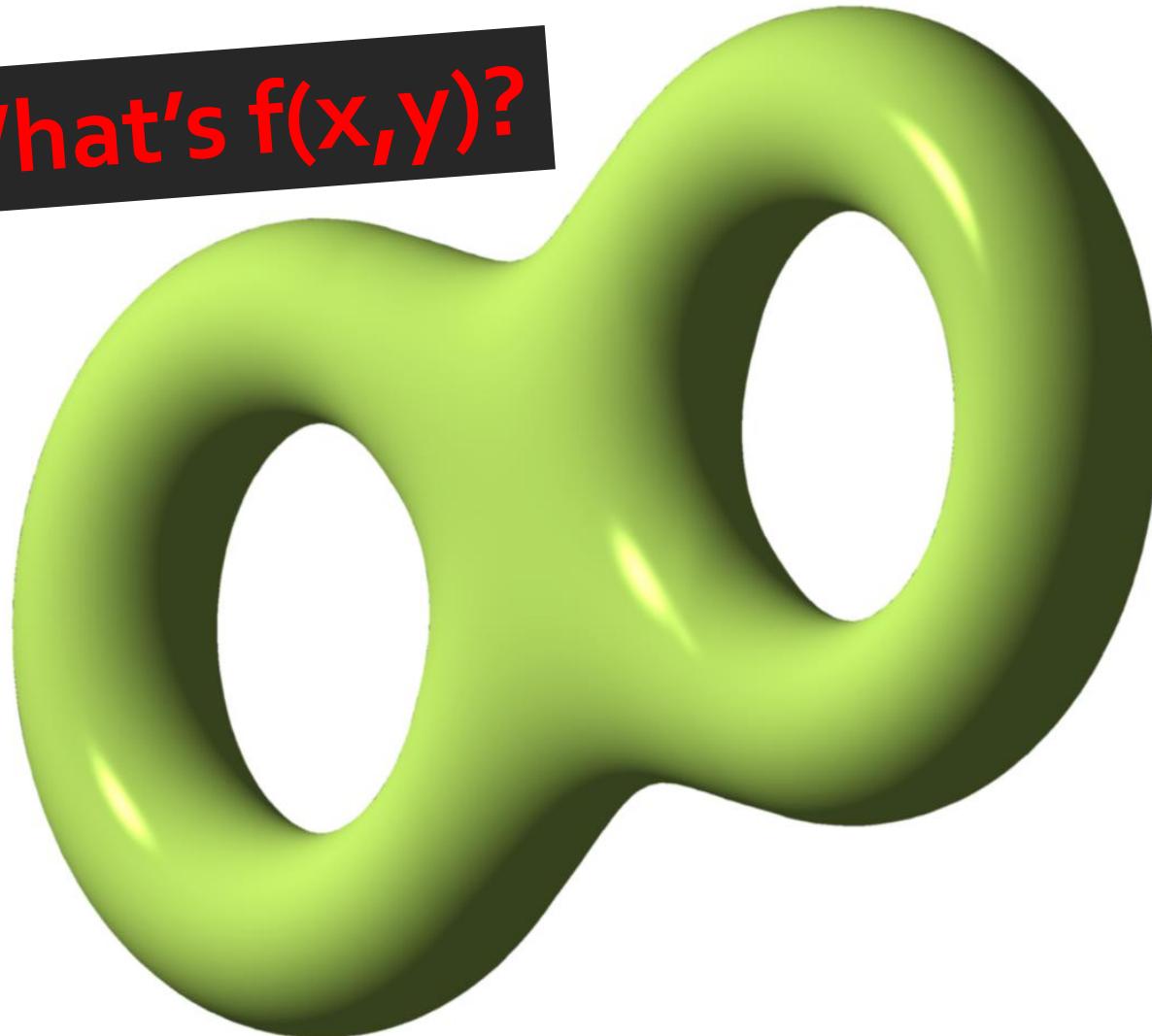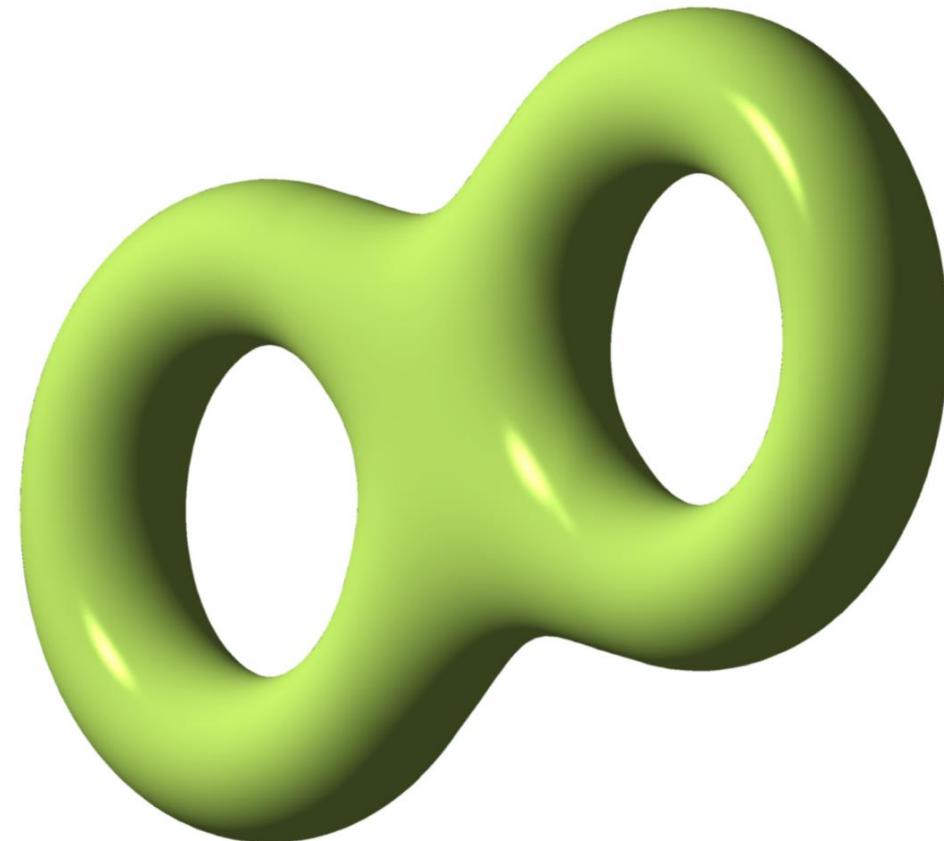
Graphs of Smooth Functions

"Cusp"

$$f(t) = (t^2, t^3)$$
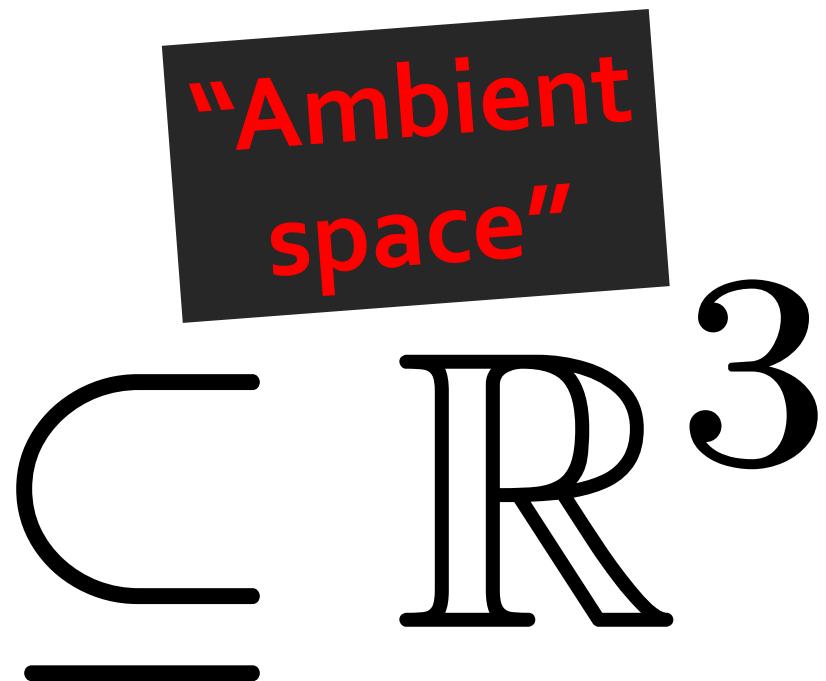
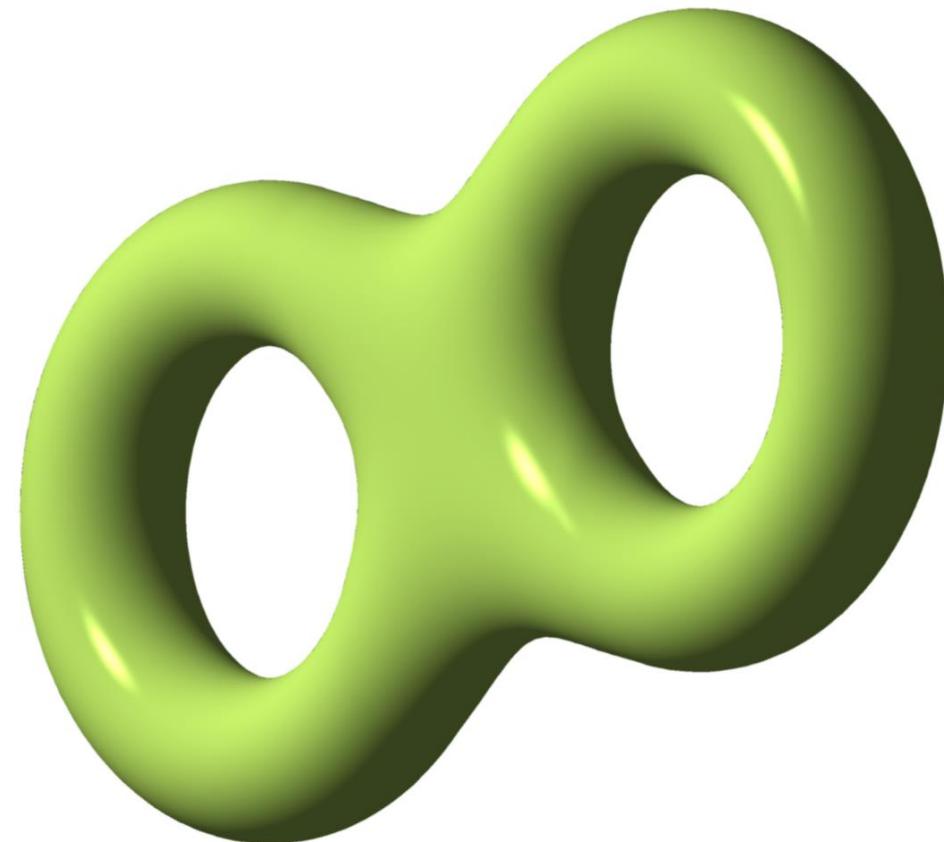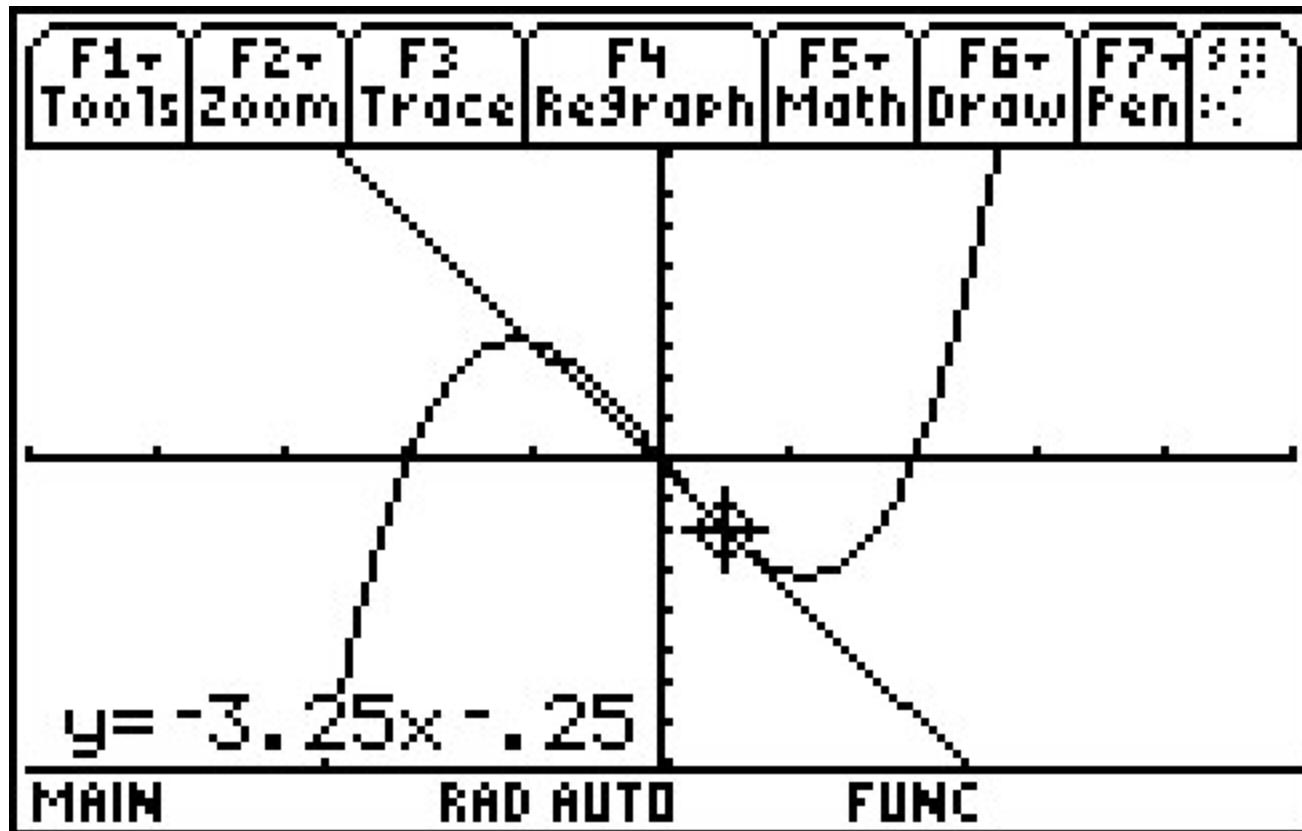# Graphs of Smooth Functions

What's f(x,y)?

What *Can* We Say?

$$\subset \mathbb{R}^3$$
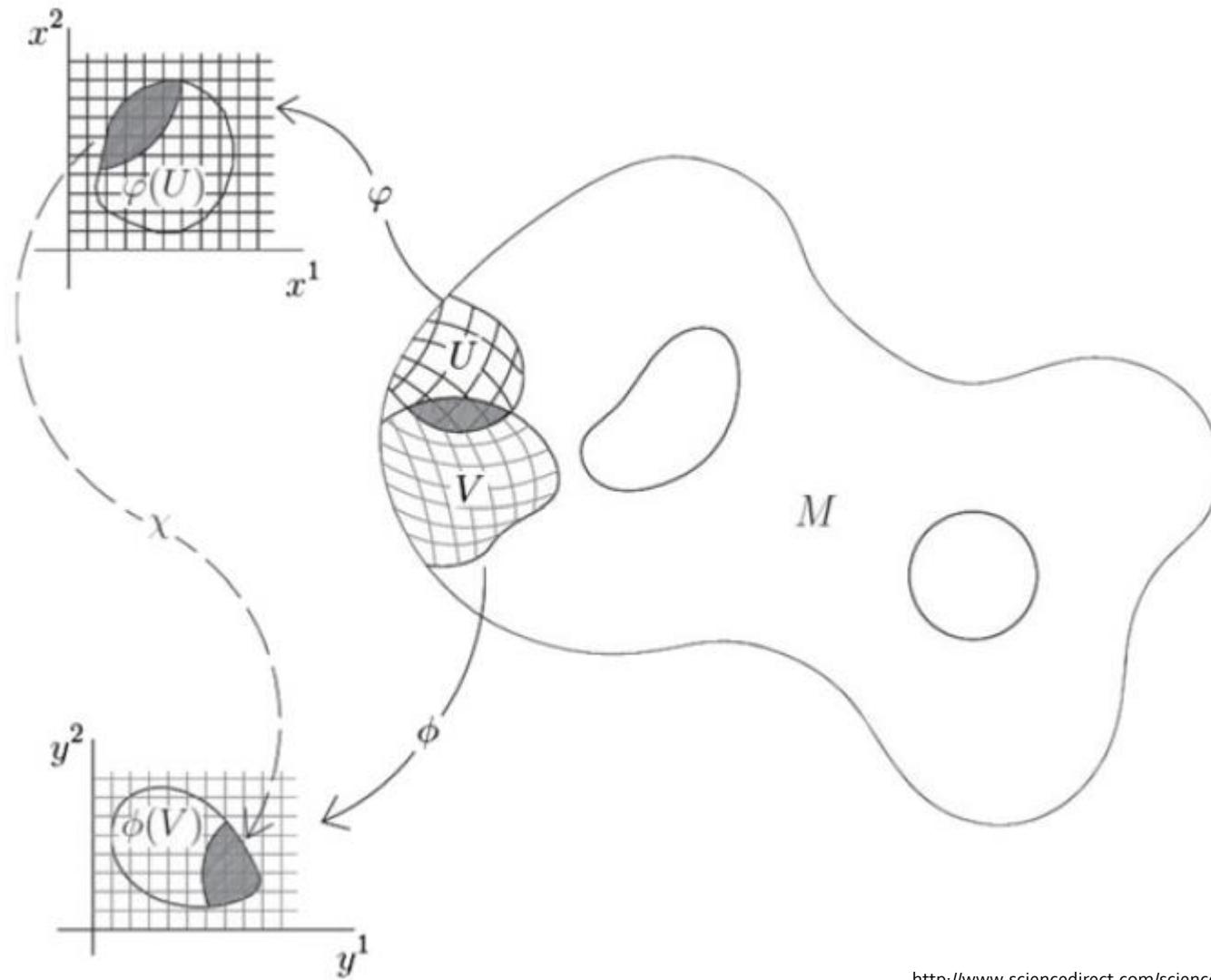
What *Can* We Say?

"Ambient space"

$$\subseteq \mathbb{R}^3$$

Another Idea from Calculus

y= -3.25x -.25

Local linearity

Coordinate Charts

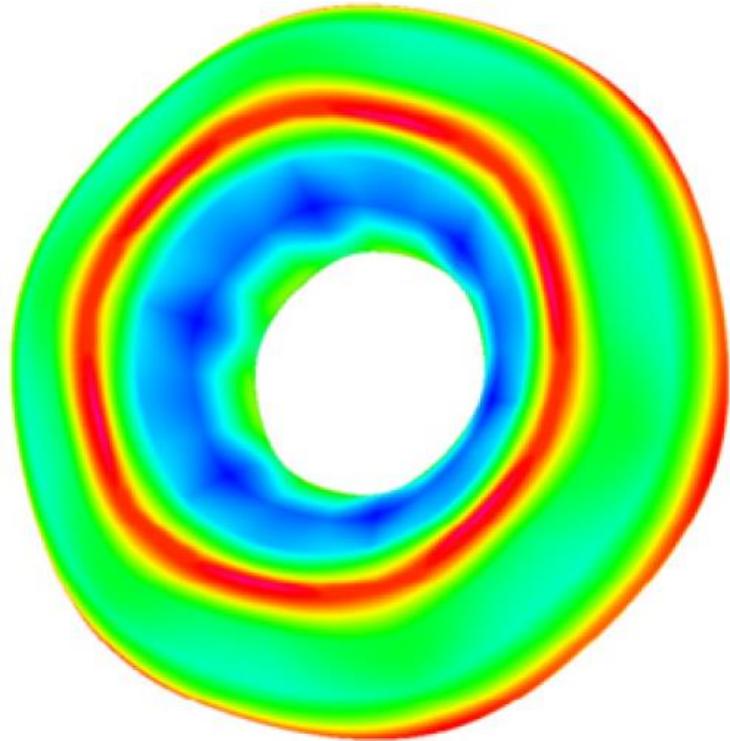$$\subseteq \mathbb{R}^3$$

$$\subseteq \mathbb{R}^2$$
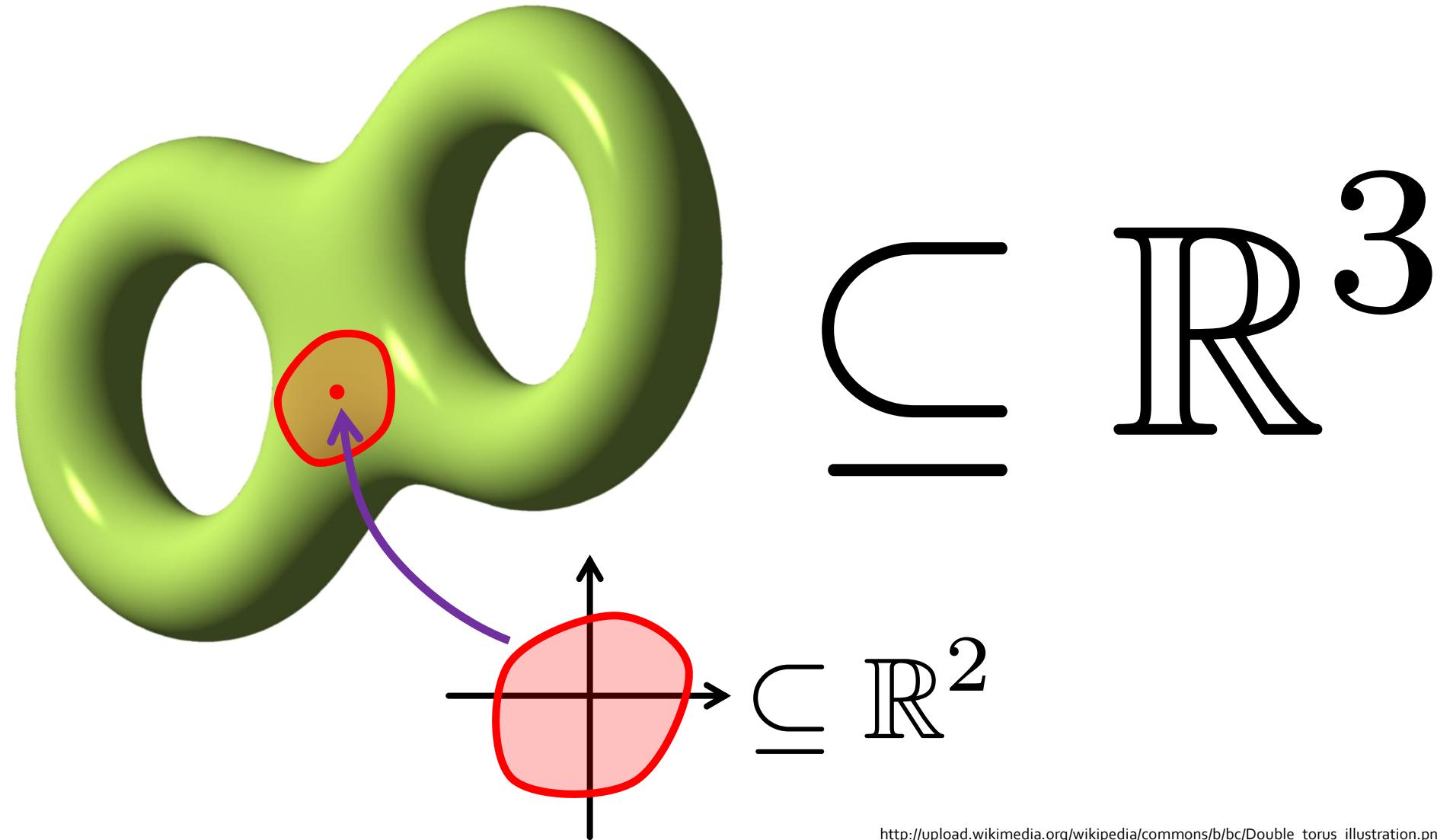
Differential Geometry Toolbox
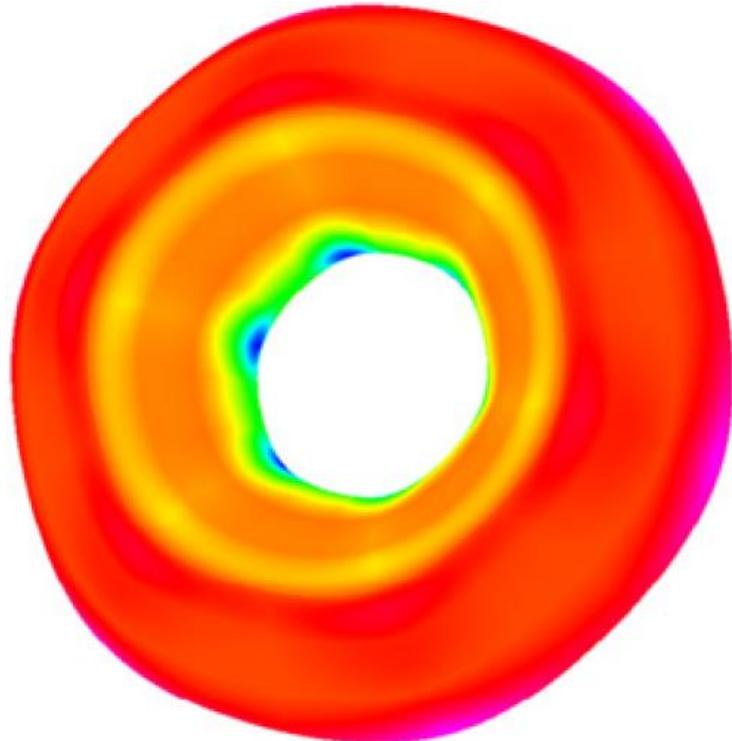
$$K = \kappa_1 \kappa_2$$

$$H = {}^1\!/_2 (\kappa_1 + \kappa_2)$$

Curvature and shape properties

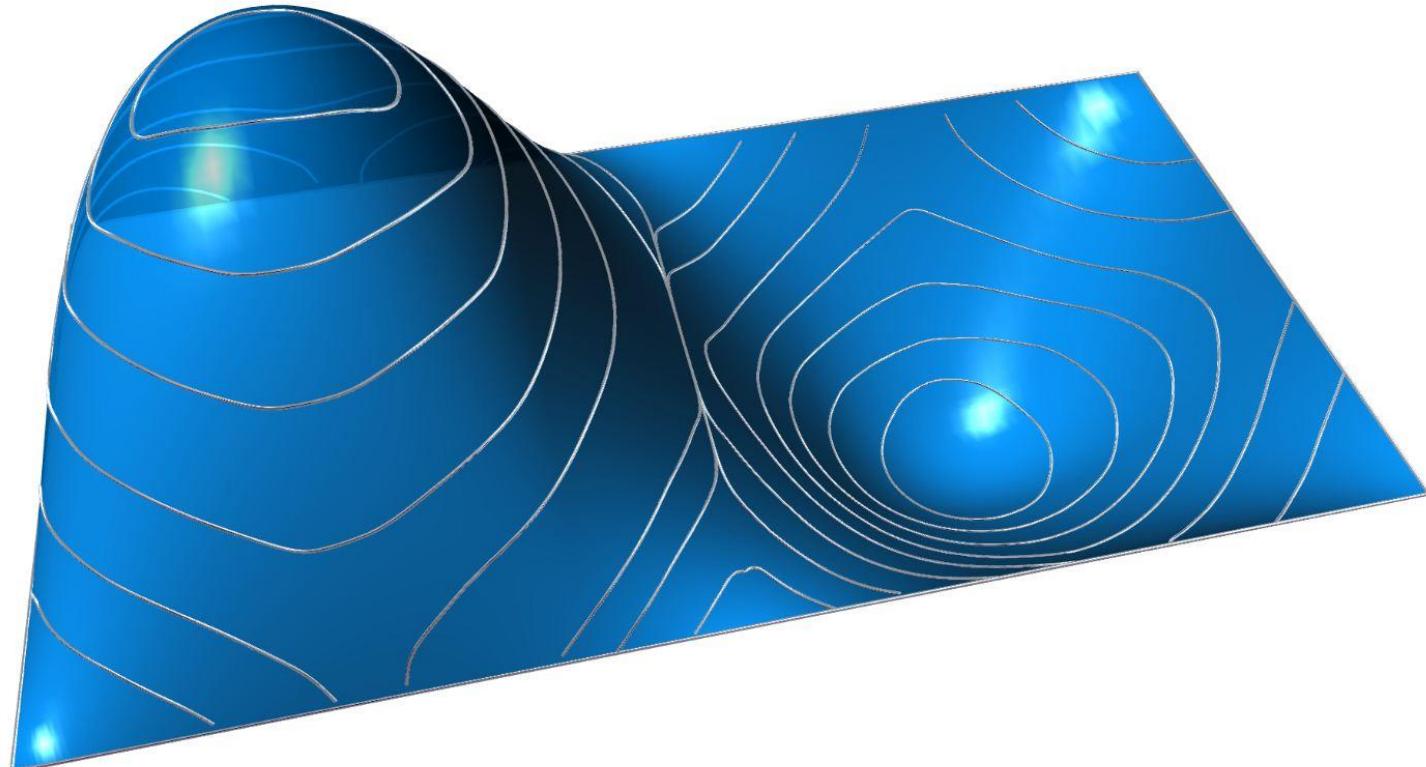# Differential Geometry Toolbox

http://users.dimi.uniud.it/~gianluca.gorni/Immagini/streamOnSurface.gif
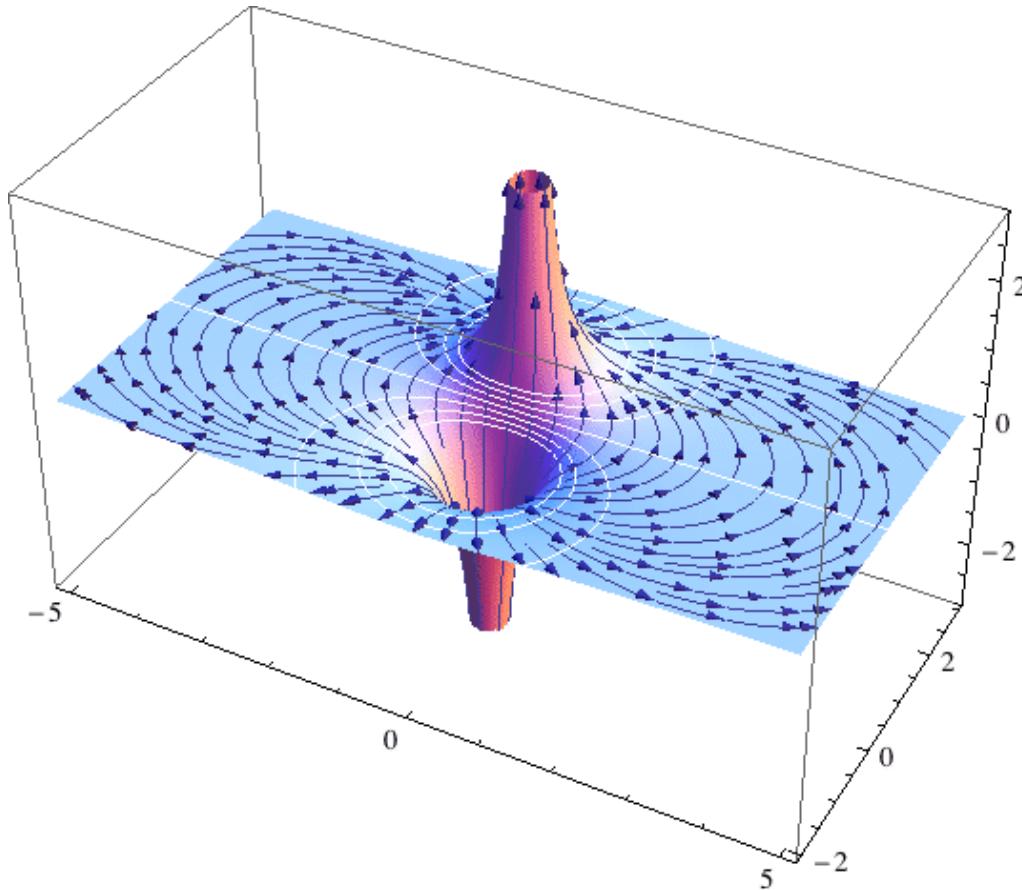
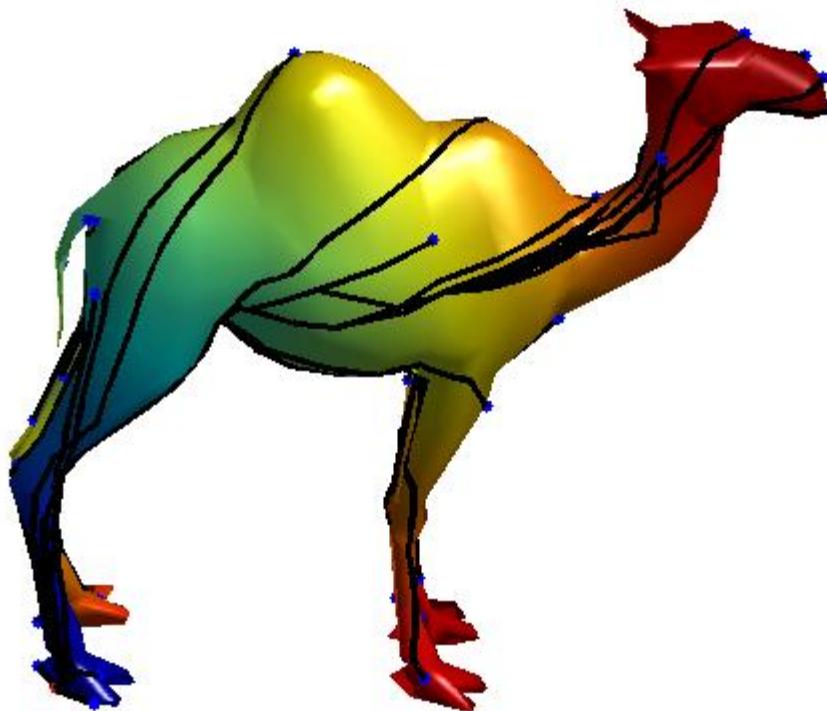# Flows and vector fields

Differential Geometry Toolbox

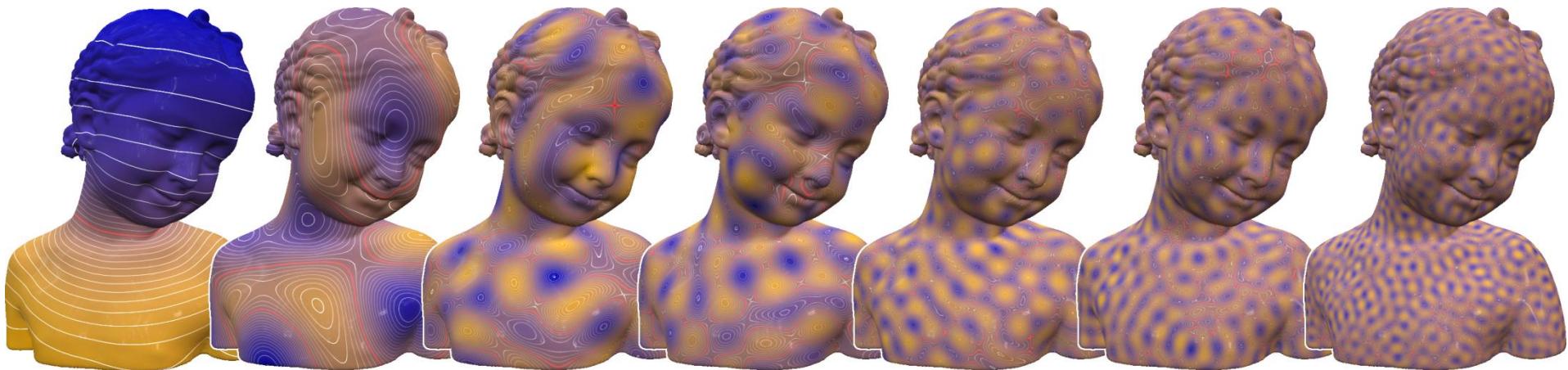## Feature points and curves

Differential Geometry Toolbox

http://www.ceremade.dauphine.fr/~peyre/numerical-tour/tours/shapes_2_bendinginv_3d/
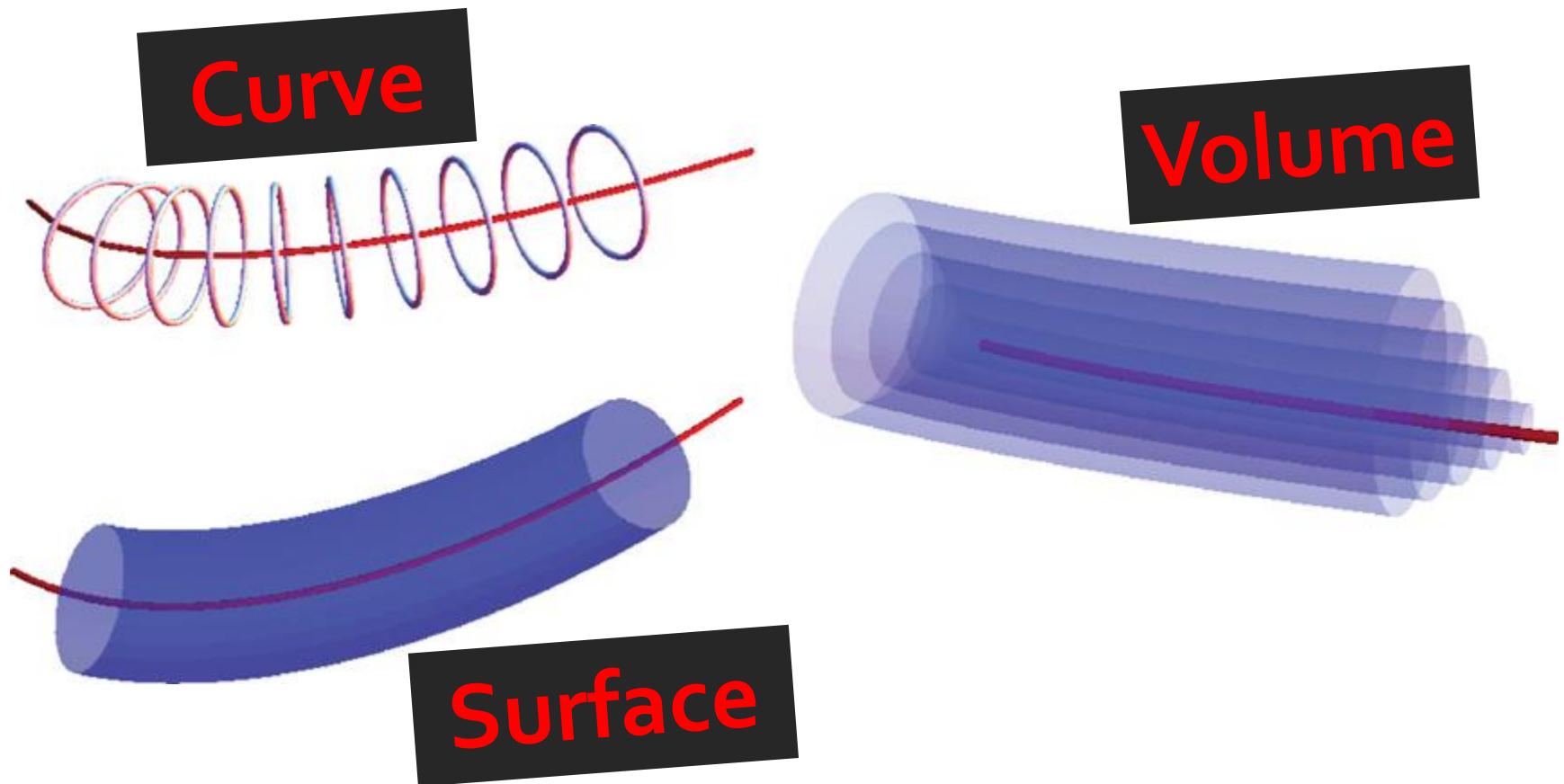
Distances

Differential Geometry Toolbox

http://alice.loria.fr/publications/papers/2008/ManifoldHarmonics//photo/bimba_mhb.png

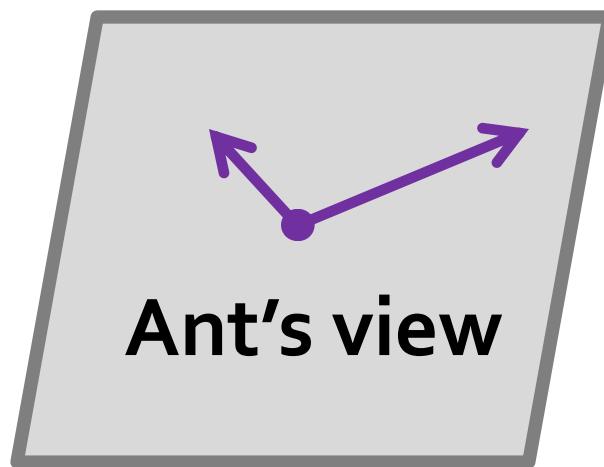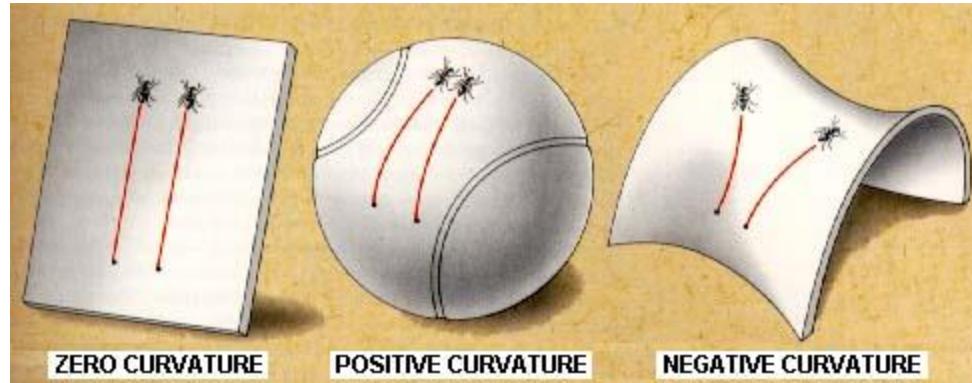## Differential equations

Generalization: Manifolds

Curve

Volume

Surface

k-dimensional object in *n* dimensions

Generalization: Riemannian

ZERO CURVATURE   POSITIVE CURVATURE   NEGATIVE CURVATURE

Ant's view

Only need angles and distances
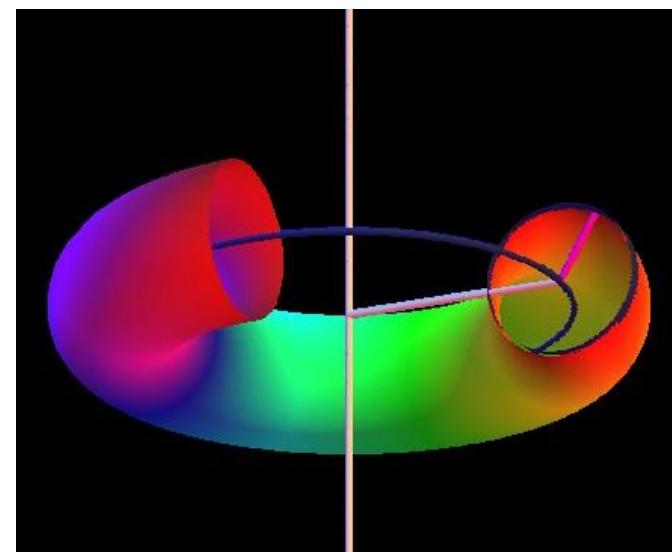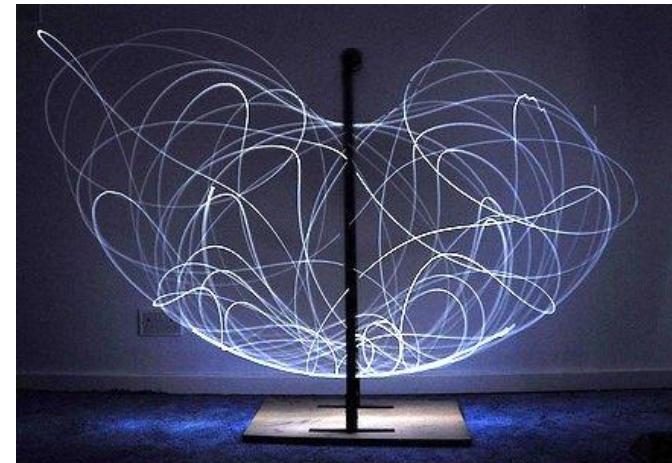
Generalization: Riemannian
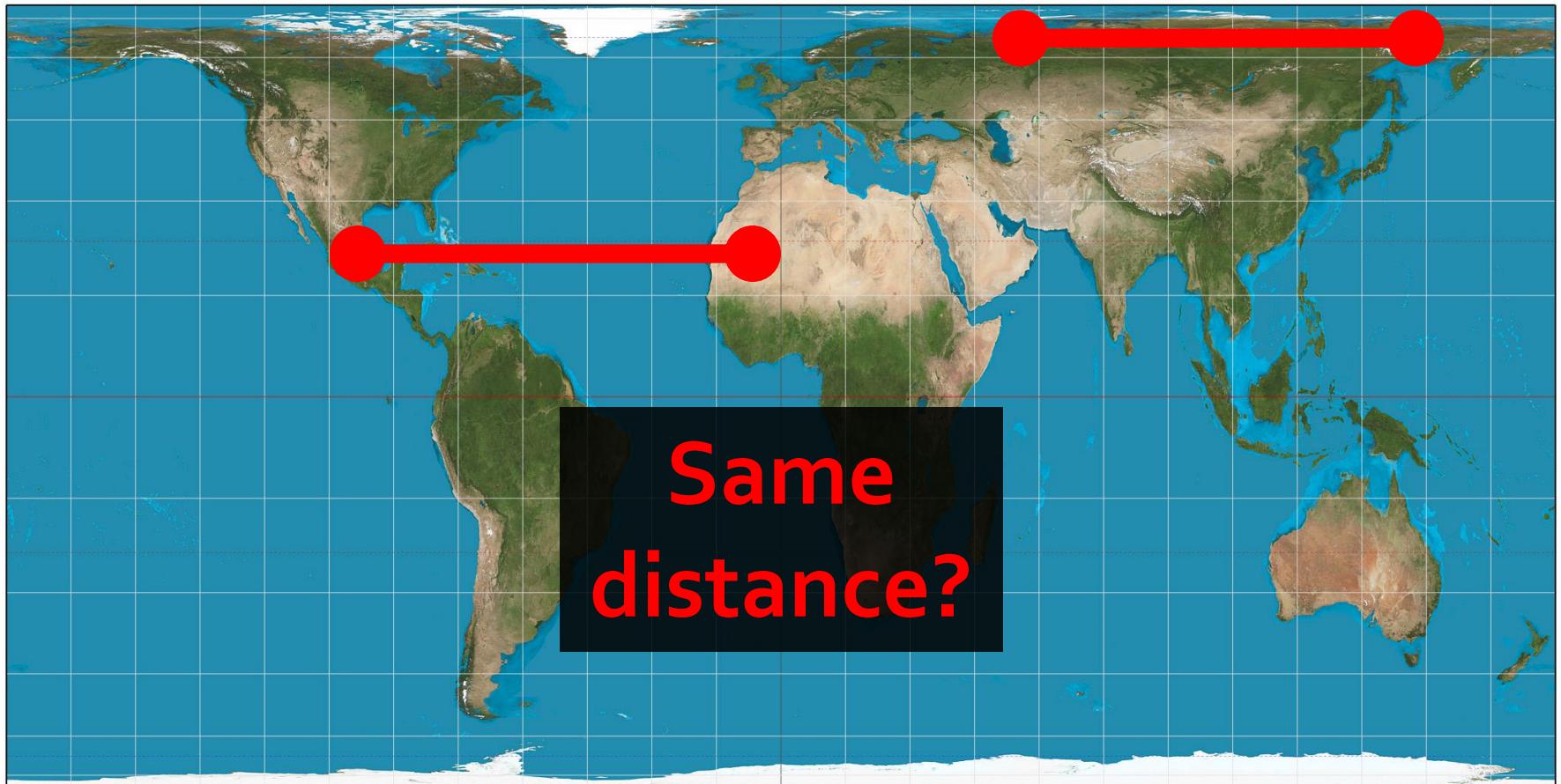
Same distance?

http://upload.wikimedia.org/wikipedia/commons/2/2c/Hobo%E2%80%93Dyer_projection_SW.jpg

Only need angles and distances

Sharp Edges, Boundary
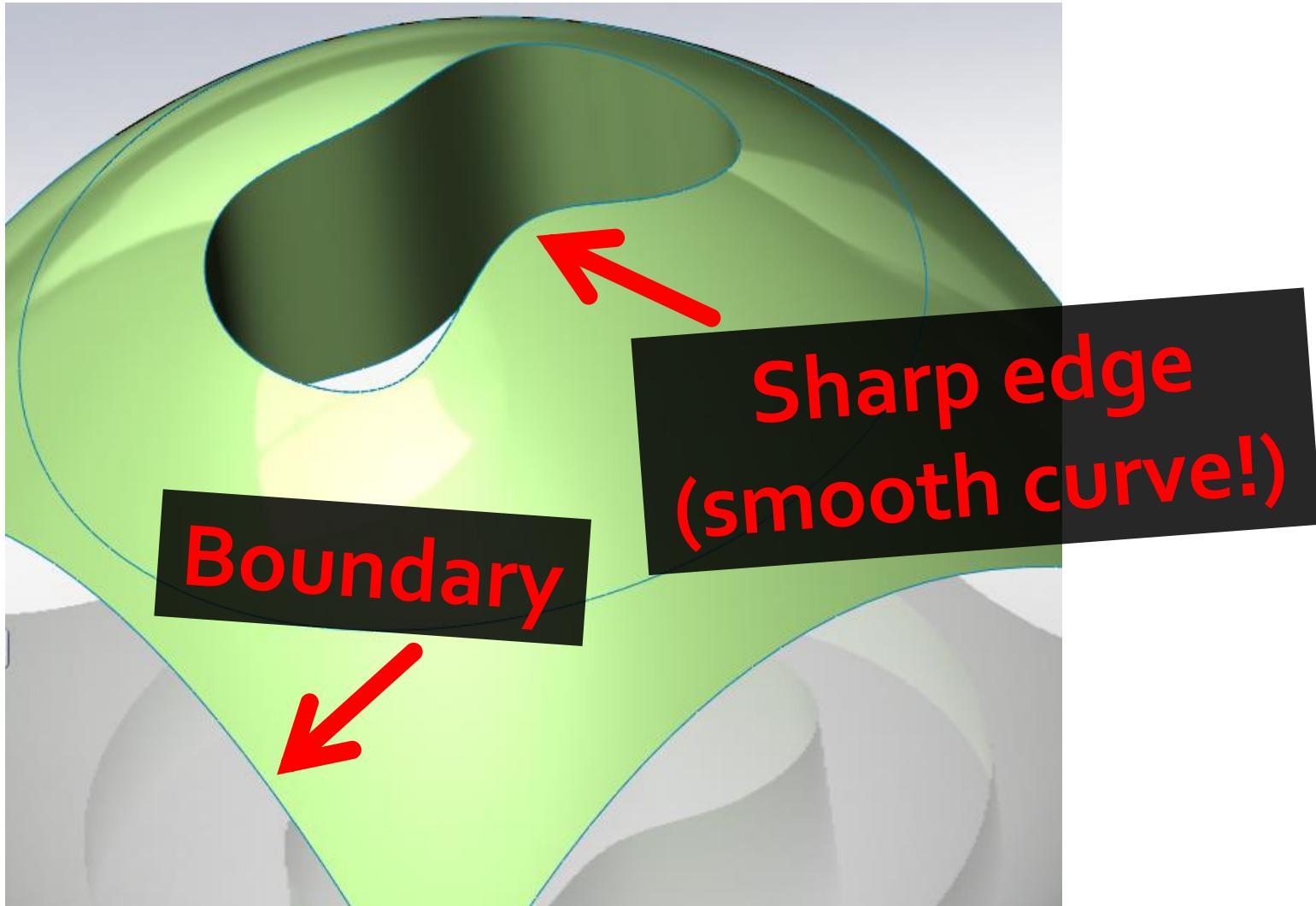
Sharp edge (smooth curve!)

Boundary

"Hidden" Geometry
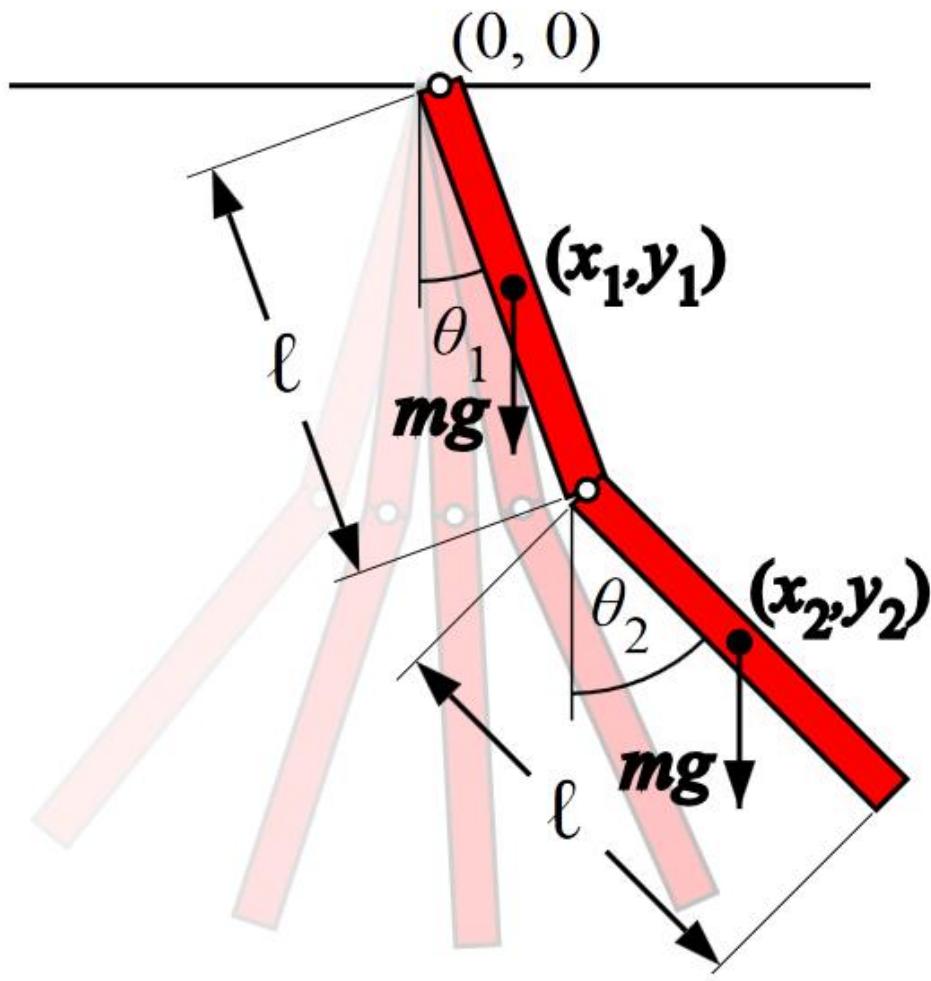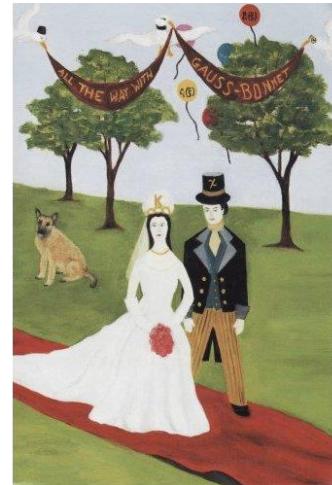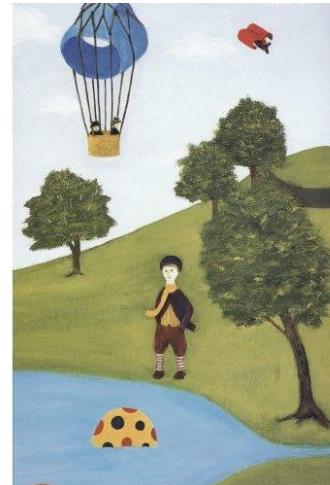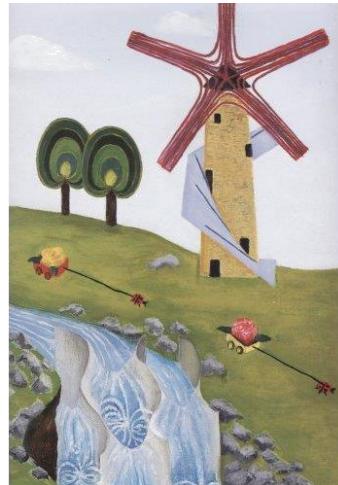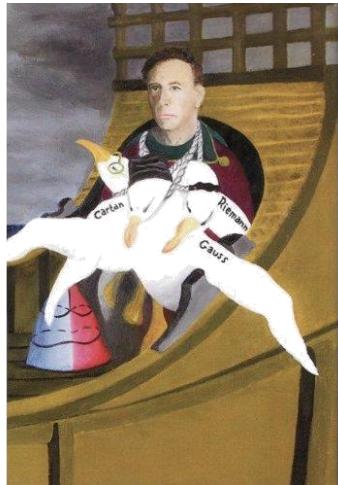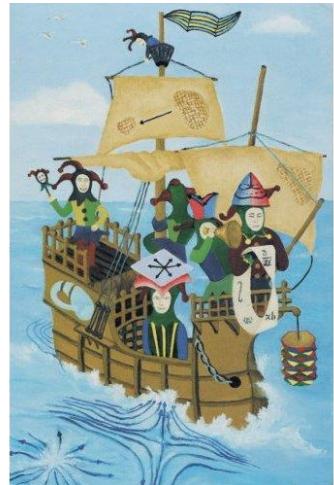
Spivak: *A Comprehensive Introduction to Differential Geometry*

1. *Continuous* differential geometry

Two Parallel Threads
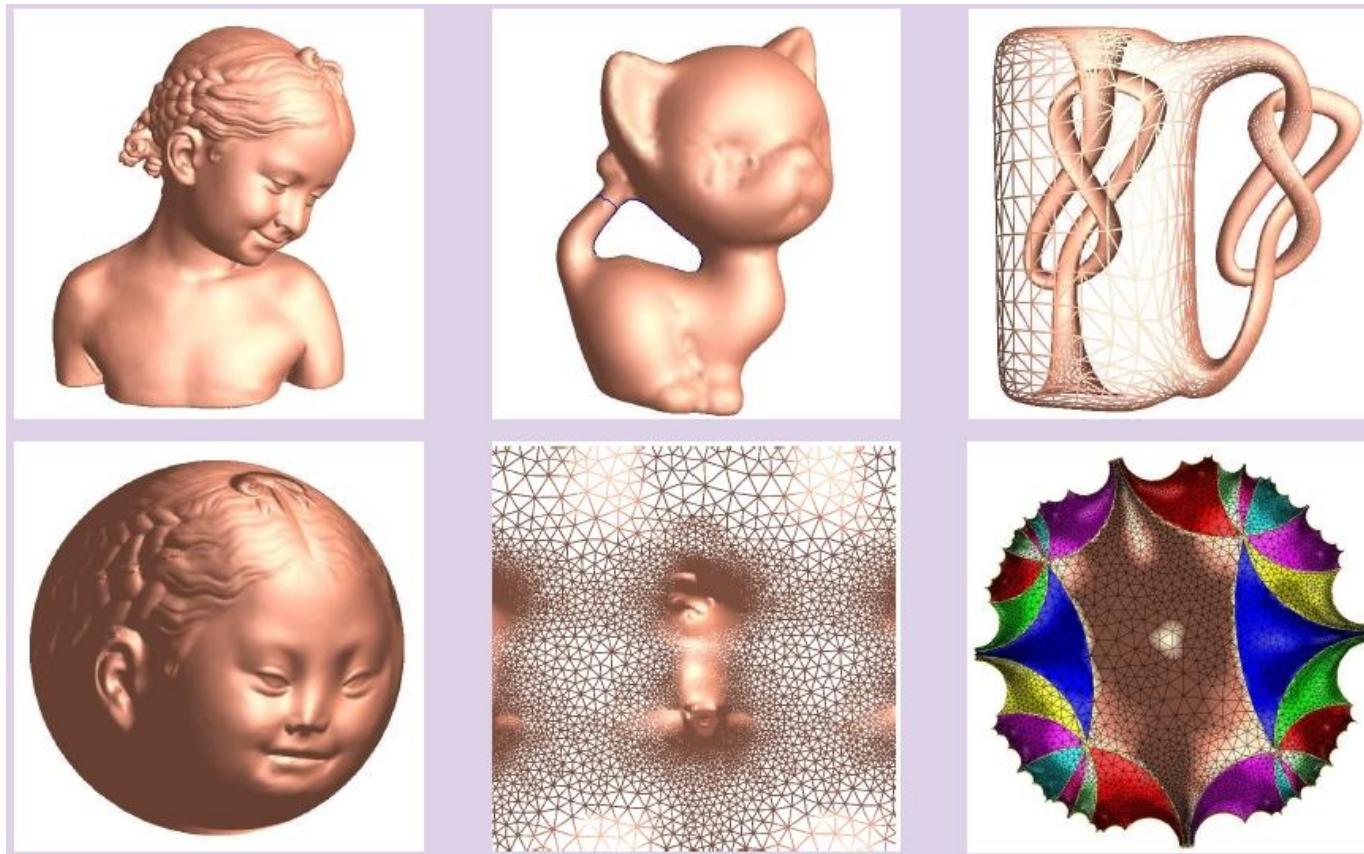
http://www.geomtop.org/teaching/conformal_2012Summer/title_image.jpg

2. *Discrete* differential geometry

Typical Geometric Input
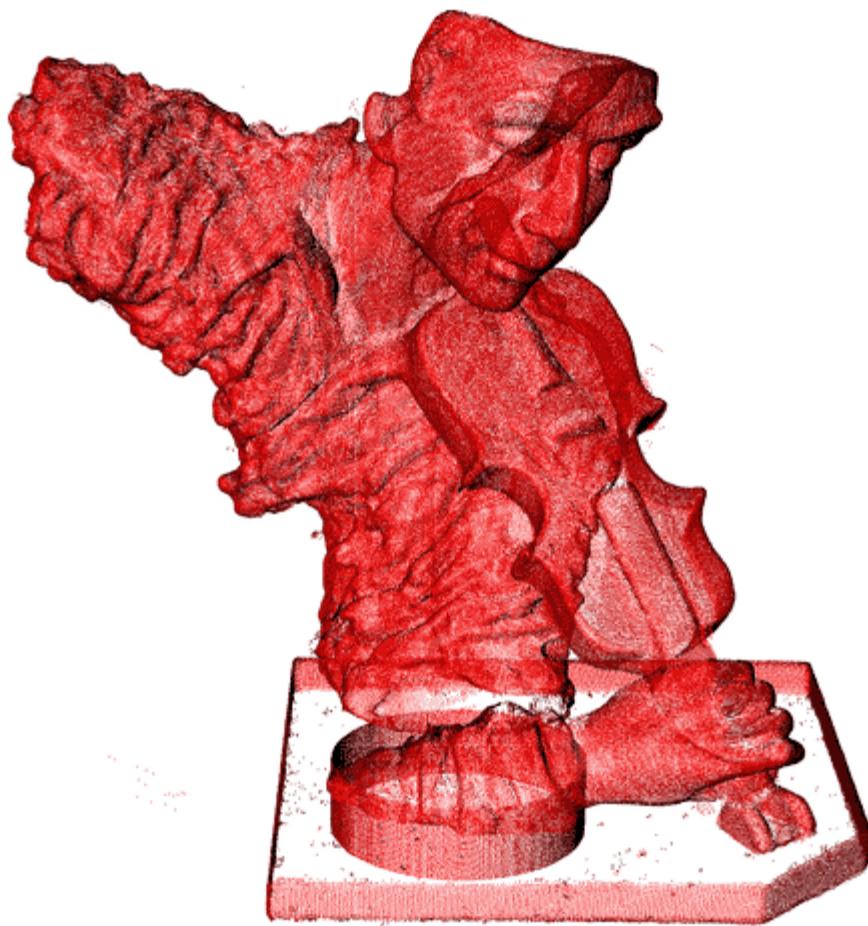
After Some Cleaning

Triangle mesh

$$M = (V, T)$$

What conditions are needed?

Triangle mesh

What is a *Discrete* Surface?
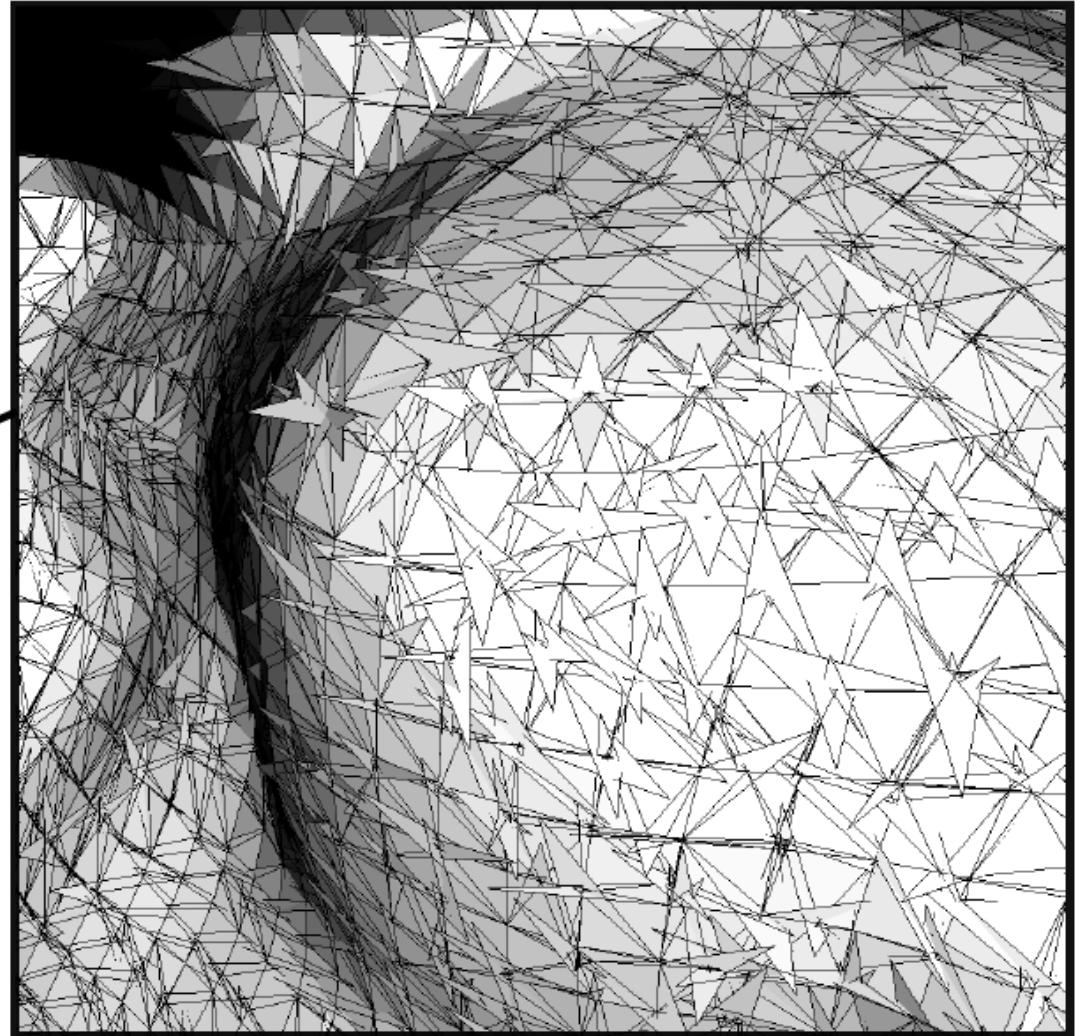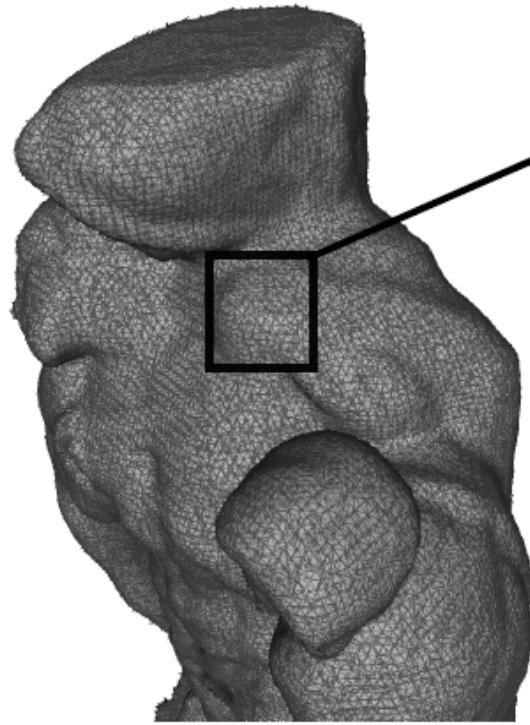
Nonmanifold Edge

Manifold Mesh

1. Each **edge** is incident to one or two faces

2. **Faces** incident to a vertex form a closed or open fan

Invalid Meshes vs. Bad Meshes

Nonuniform
areas and **angles**

Why is Meshing an Issue?

How to you interpret
one value per vertex?

Convergence Analysis of Discrete Differential Geometry Operators over Surfaces

Zhiqiang Xu[1], Guoliang Xu[2], and Jia-Guang Sun[3]

[1] Department of Computer Science, Tsinghua University, Beijing 100084, China
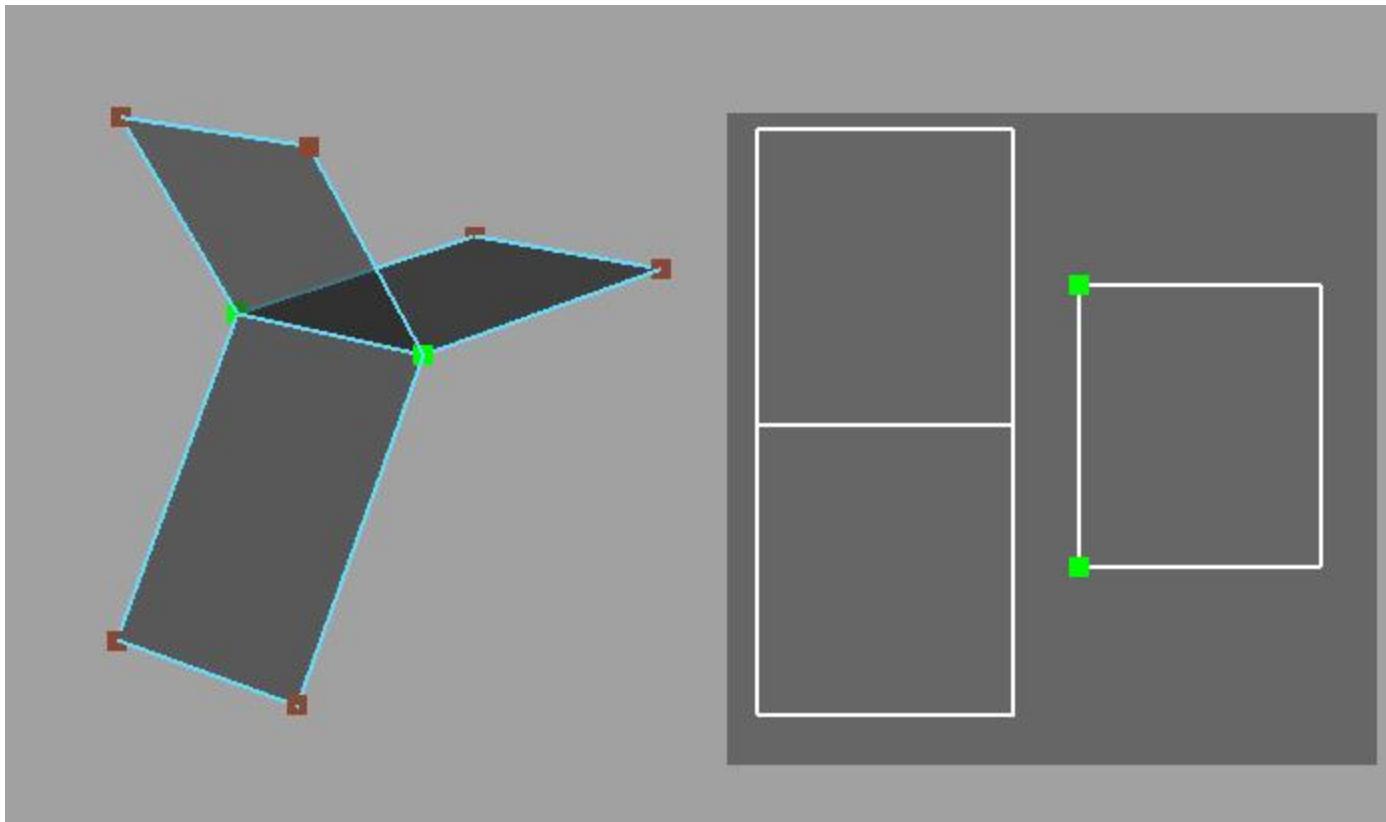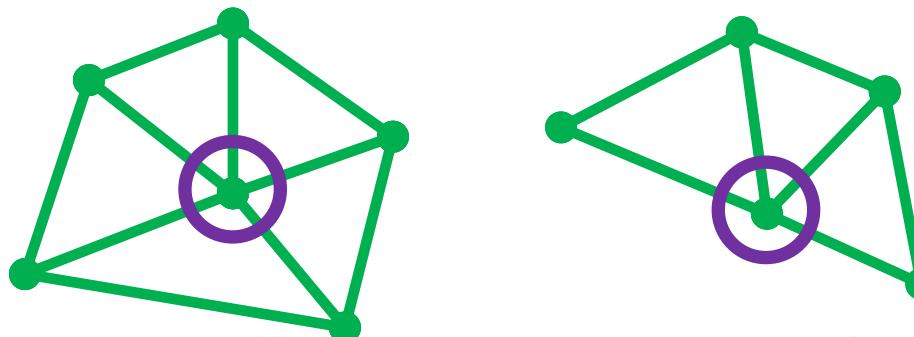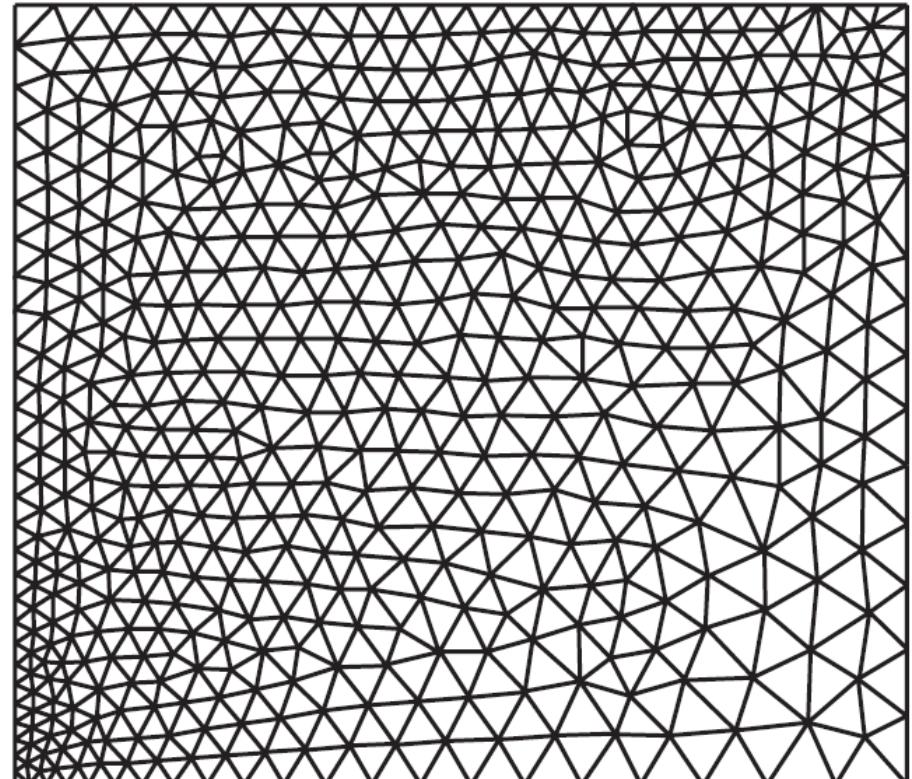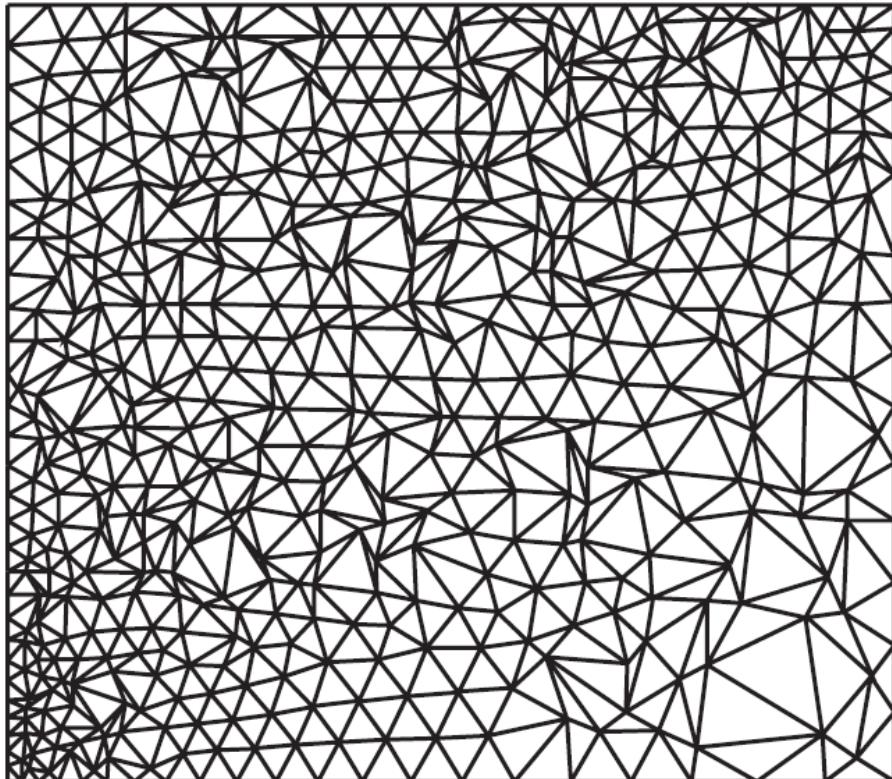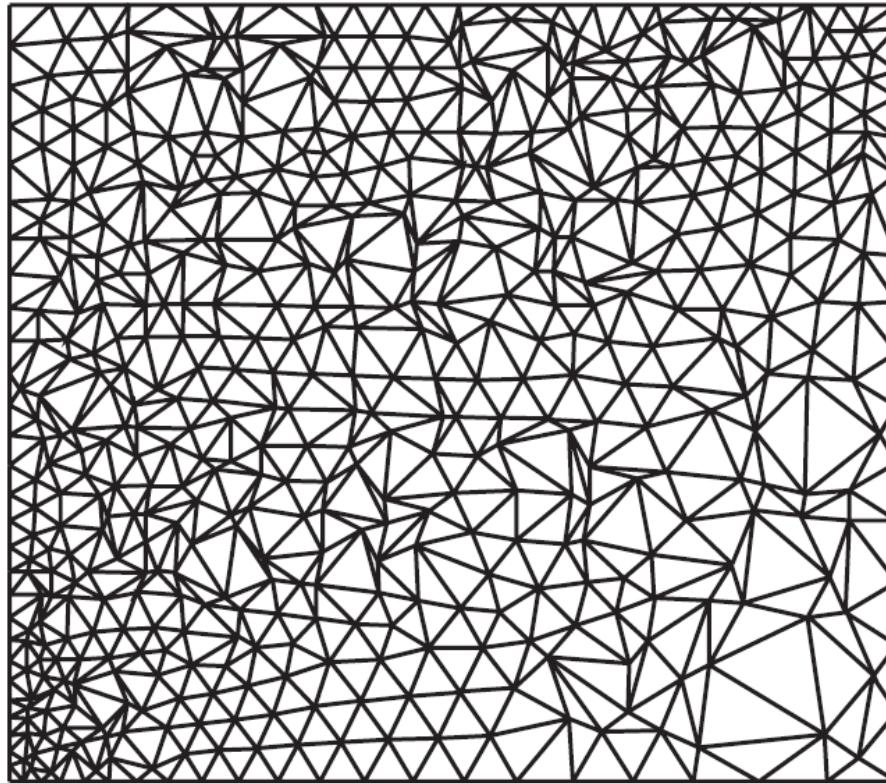xuzq@tsinghua.edu.cn
[2] ICMSEC, LSEC, Academy of Mathematics and System Sciences,
Chinese Academy of Sciences, Beijing, China
xuguo@lsec.cc.ac.cn
[3] School of Software, Tsinghua University, Beijing 100084, China
sunjg@tsinghua.edu.cn

Abstract. In this paper, we study the convergence property of several discrete schemes of the surface normal. We show that the arithmetic mean, area-weighted averaging, and angle-weighted averaging schemes have quadratic convergence rate for a special triangulation scenario of the surfaces. By constructing a counterexample, we also show that it is impossible to find a discrete scheme of normals that has quadratic convergence rate over any triangulated surface and hence give a negative answer for the open question raised by D.S.Meek and D.J. Walton. More-over, we point out that one cannot build a discrete scheme for Gaussian curvature, mean curvature and Laplace-Beltrami operator that converges over any triangulated surface.

Continuous Connection

Convergence Analysis of Discrete Differential Geometry Operators over Surfaces

Zhiqiang Xu[1], Guoliang Xu[2], and Jia-Guang Sun[3]

[1] Department of Computer Science, Tsinghua University, Beijing 100084, China
xuzq@tsinghua.edu.cn
[2] ICMSEC, LSEC, Academy of Mathematics and System Sciences,

sunjg@tsinghua.edu.cn

In this paper, we study the convergence property of several discrete schemes of the surface normal.

Abstract. In this paper, we study the convergence property of several

More-over, we point out that one cannot build a discrete scheme for Gaussian curvature, mean curvature and Laplace-Beltrami operator that converges over any triangulated surface.

over, we point out that one cannot build a discrete scheme for Gaussian curvature, mean curvature and Laplace-Beltrami operator that converges over any triangulated surface.

Modern Approach

Discrete

vs.
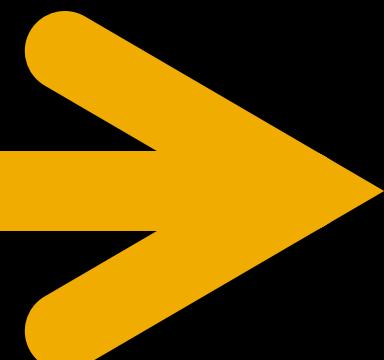
Discretized

Discrete Differential Geometry

Discrete theory *paralleling* differential geometry.

Structure preservation

[**struhk**-cher pre-zur-**vey**-sh*uh* n]:

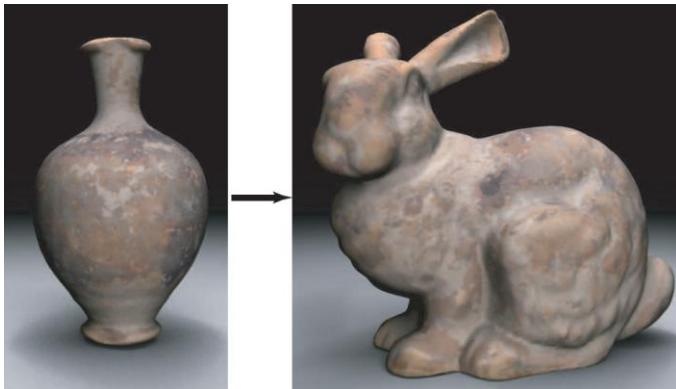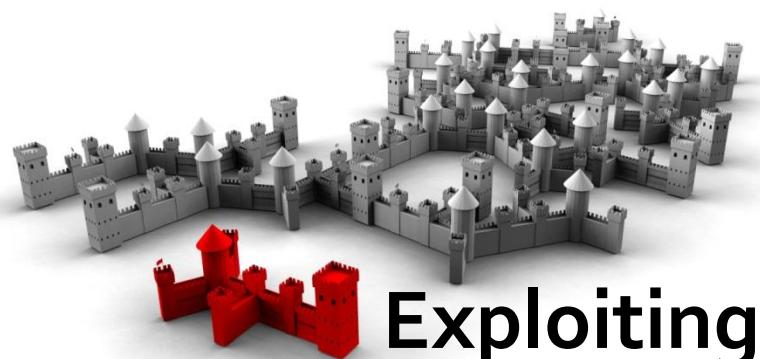Keeping properties from the continuous abstraction exactly true in a discretization.

Discrete Differential Geometry

Discrete theory paralleling differential geometry.

More in Lecture 3 and beyond.

Applications

Transfer

Retrieval

Editing

Exploiting patterns
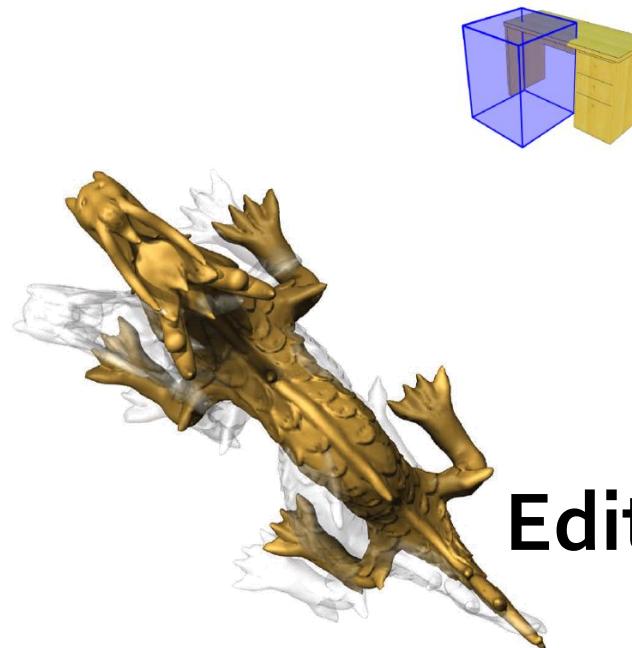
http://people.csail.mit.edu/tmertens/papers/textransfer_electronic.pdf
http://graphics.stanford.edu/~mdfisher/Data/Context.pdf
http://graphics.stanford.edu/~niloy/research/symmetrization/paper_docs/symmetrization_sig_07.pdf
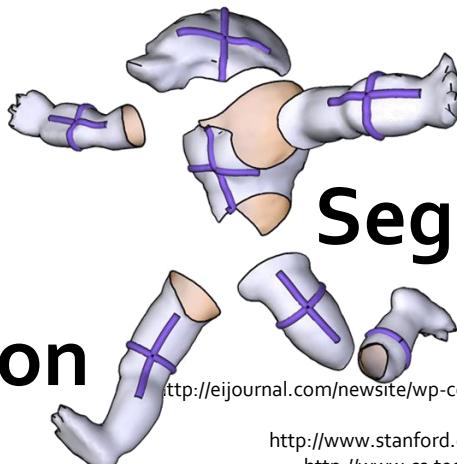http://www.mpi-inf.mpg.de/~mbokeloh/project_dockingSites.html

Graphics

Applications
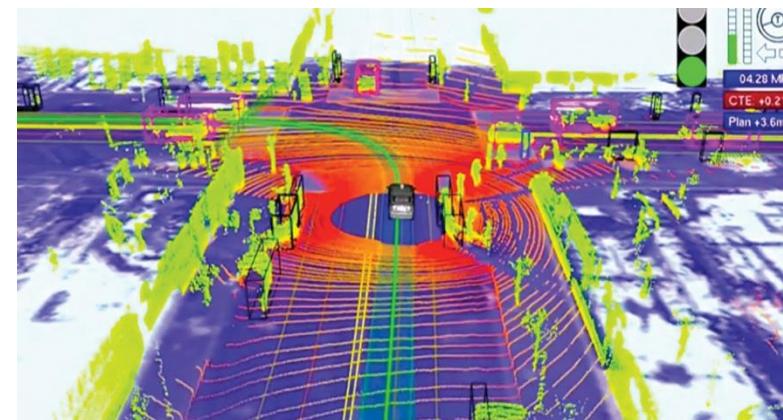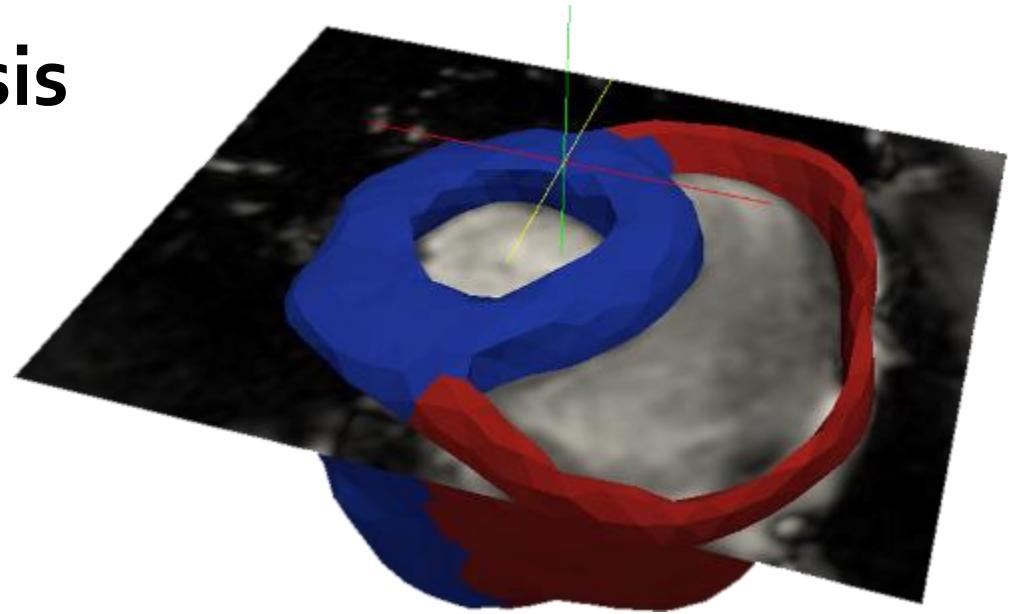

Recognition

Navigation

Reconstruction

Segmentation

Vision
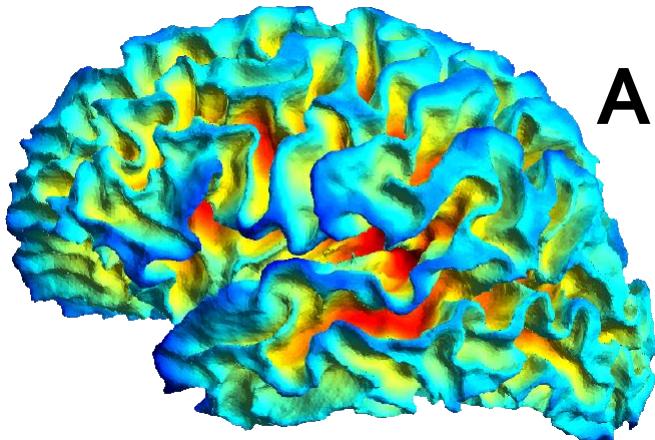
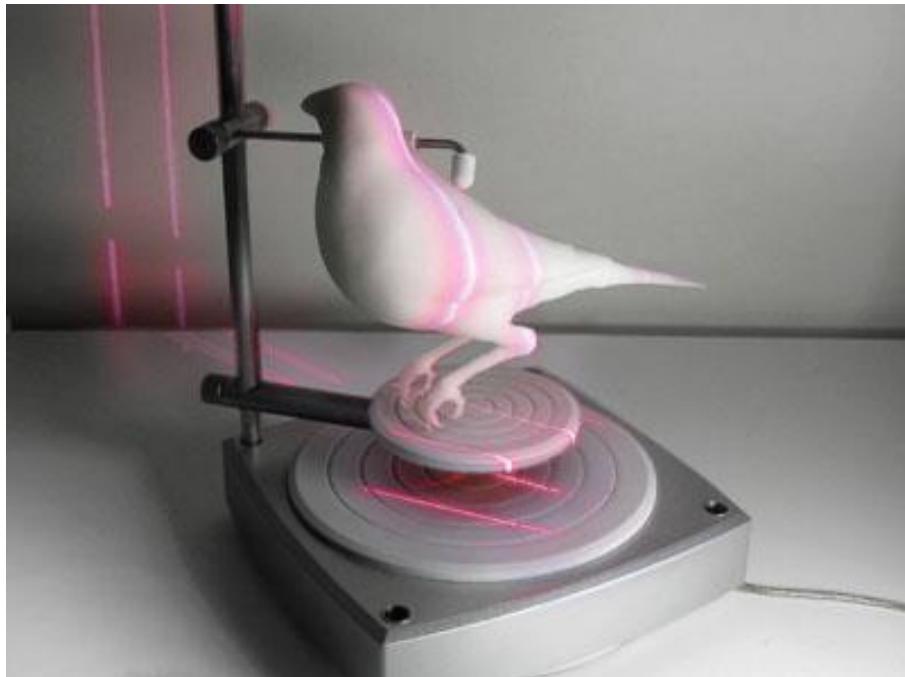# Applications

Analysis

Segmentation
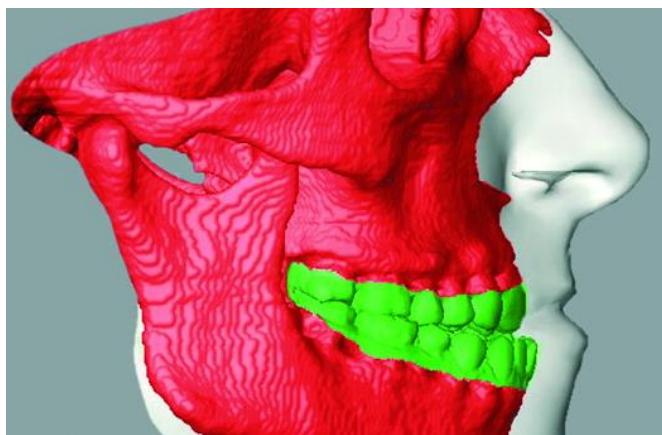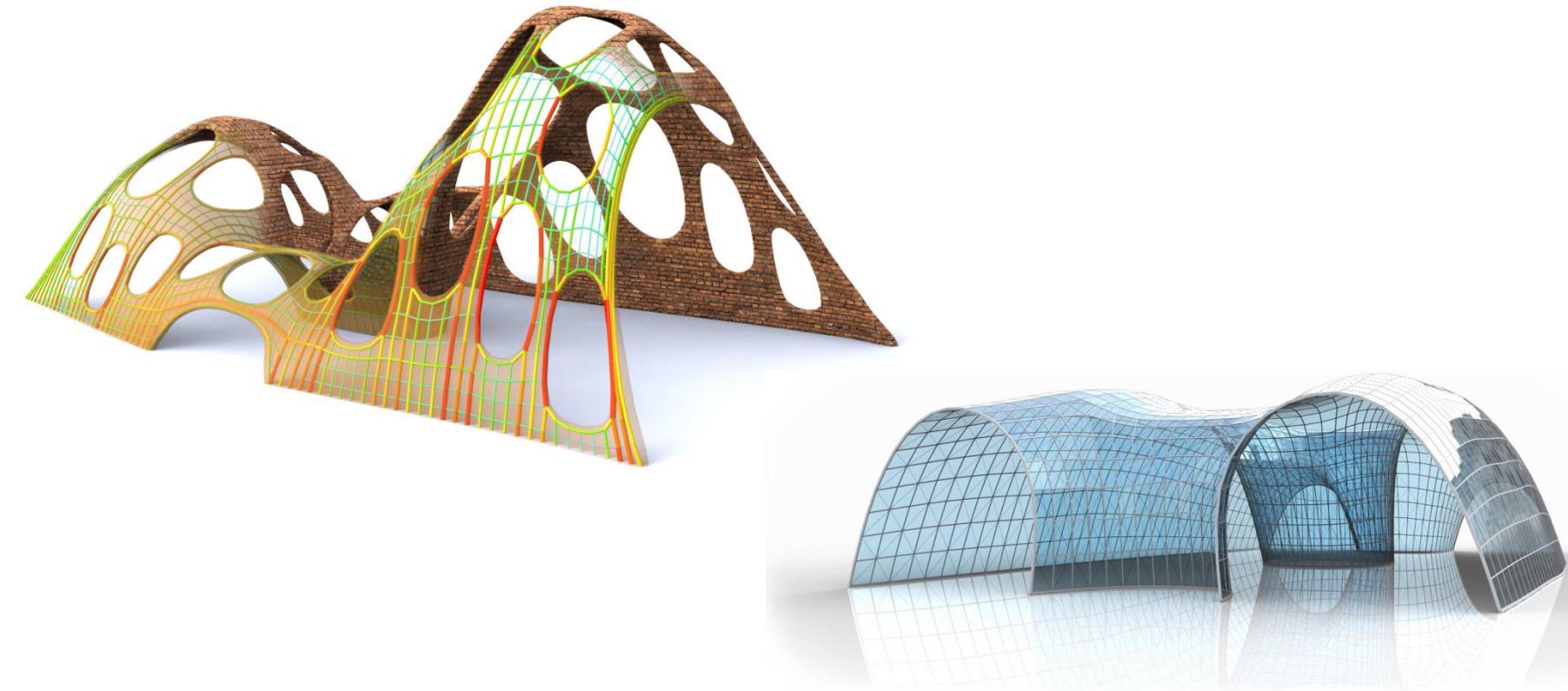
Registration

Medical Imaging

Applications

Scanning
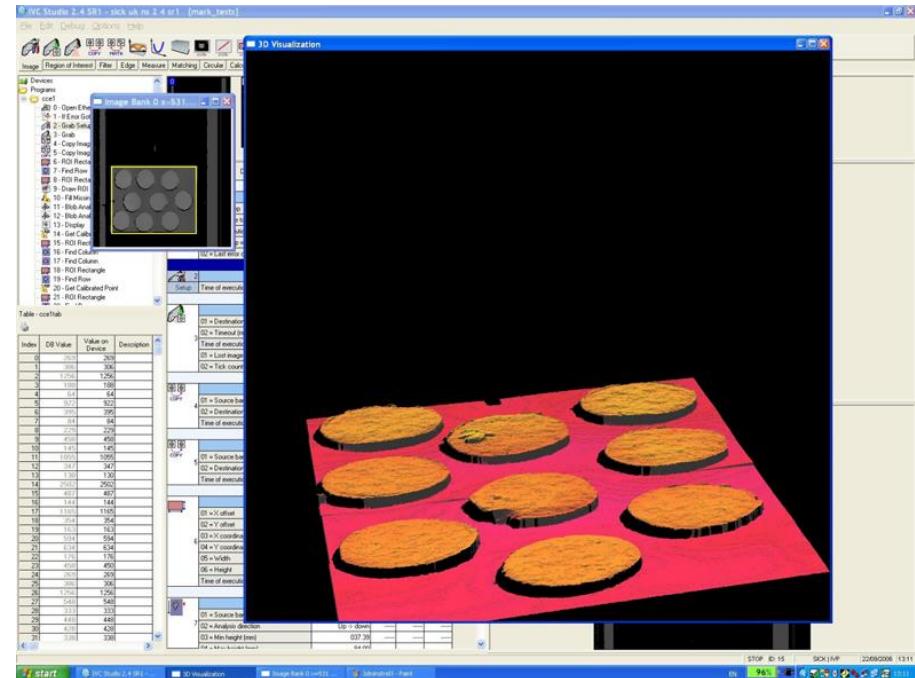
Defect detection

Manufacturing
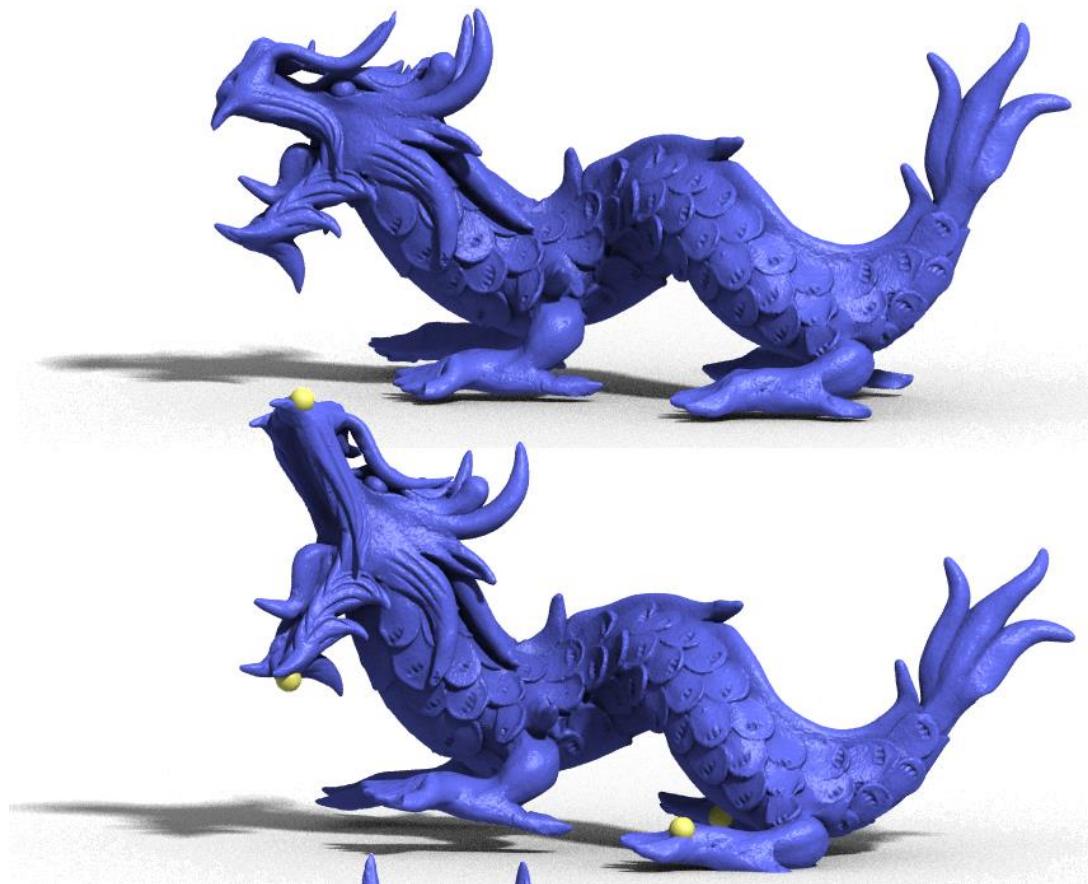
Applications

Design and analysis

http://gmsv.kaust.edu.sa/people/faculty/pottmann/pottmann_pdf/selfsupporting.pdf
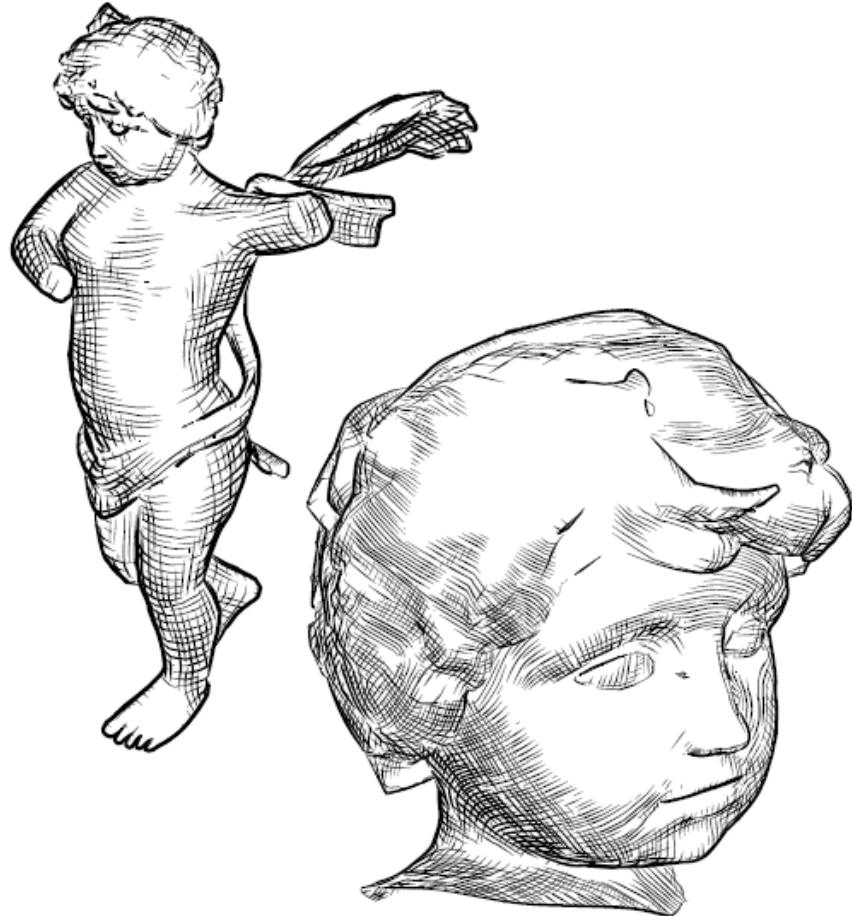
Architecture

Example Tasks

http://josiahmanson.com/joe/research/hierarchical_deform/figures/dragon_deform.png
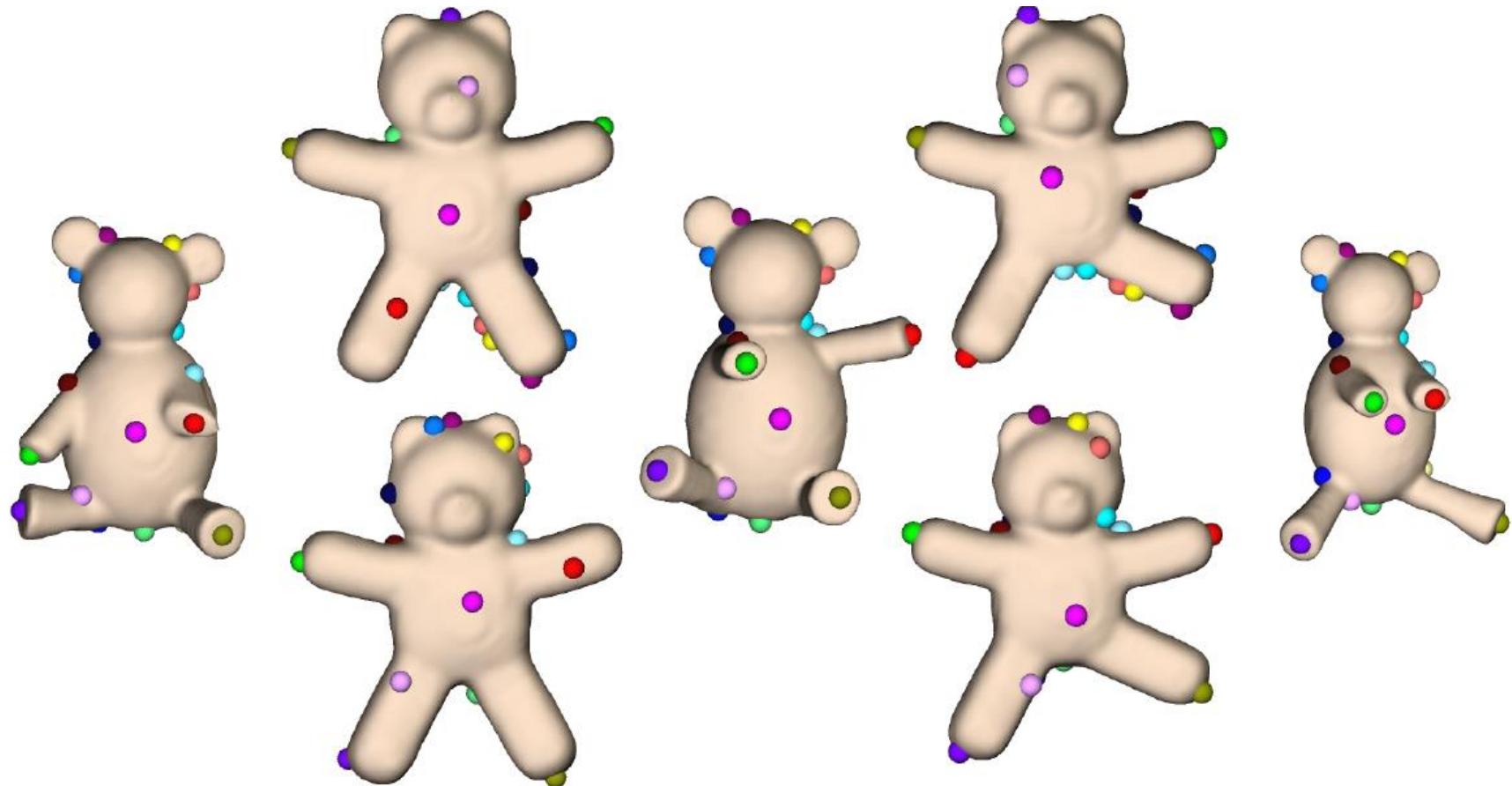
Deformation

Example Tasks

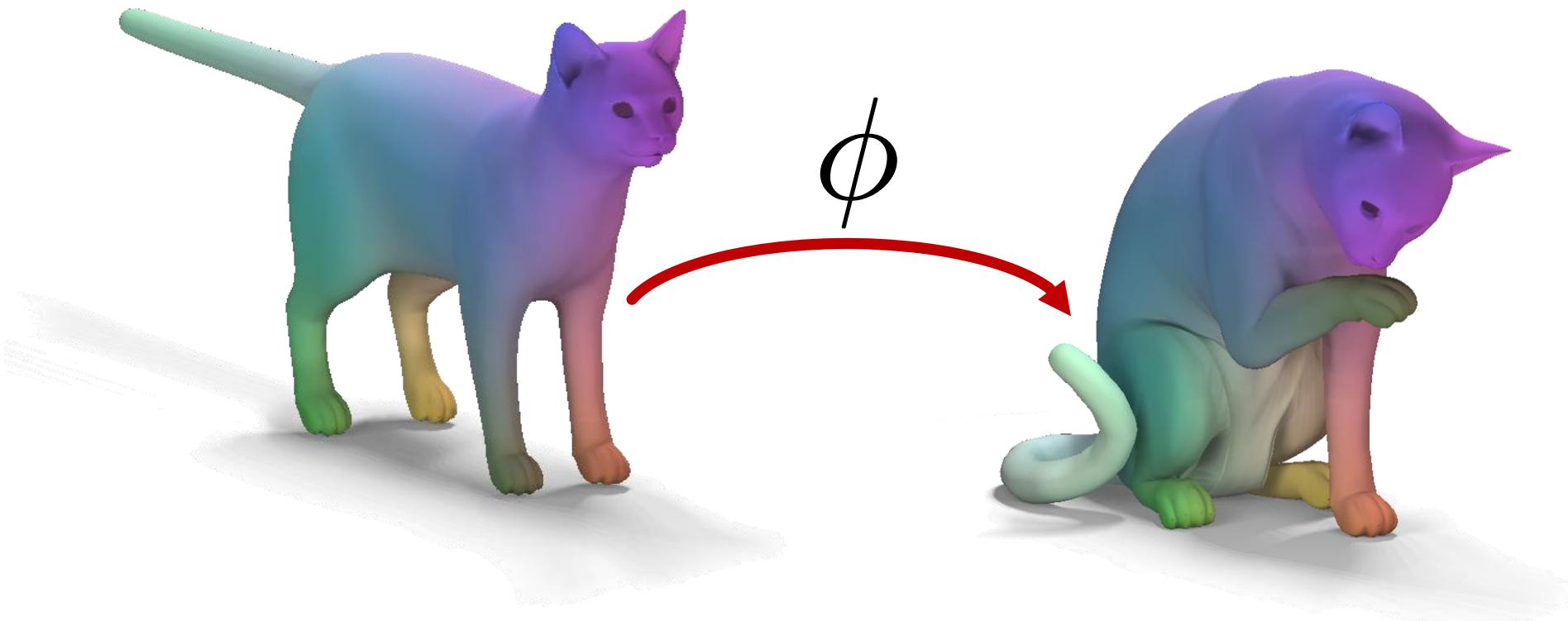http://www.cs.ucdavis.edu/~ma/SIGGRAPH02/course23/notes/S02c23_3.pdf

Expressive rendering

Example Tasks

http://graphics.stanford.edu/projects/lgl/papers/nbwyg-oaicsm-11/nbwyg-oaicsm-11.pdf
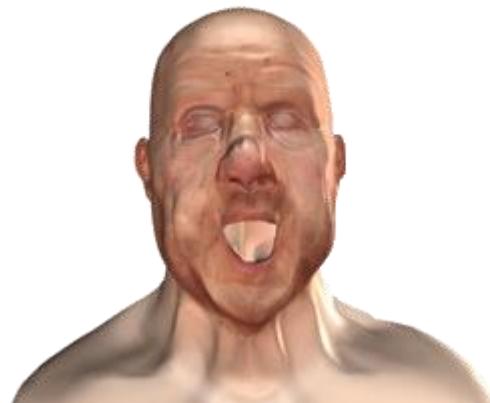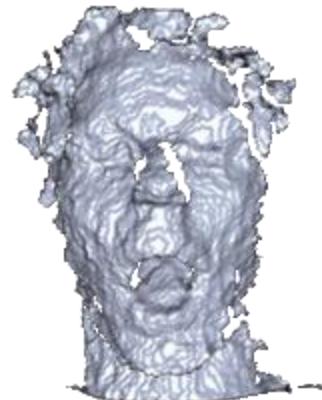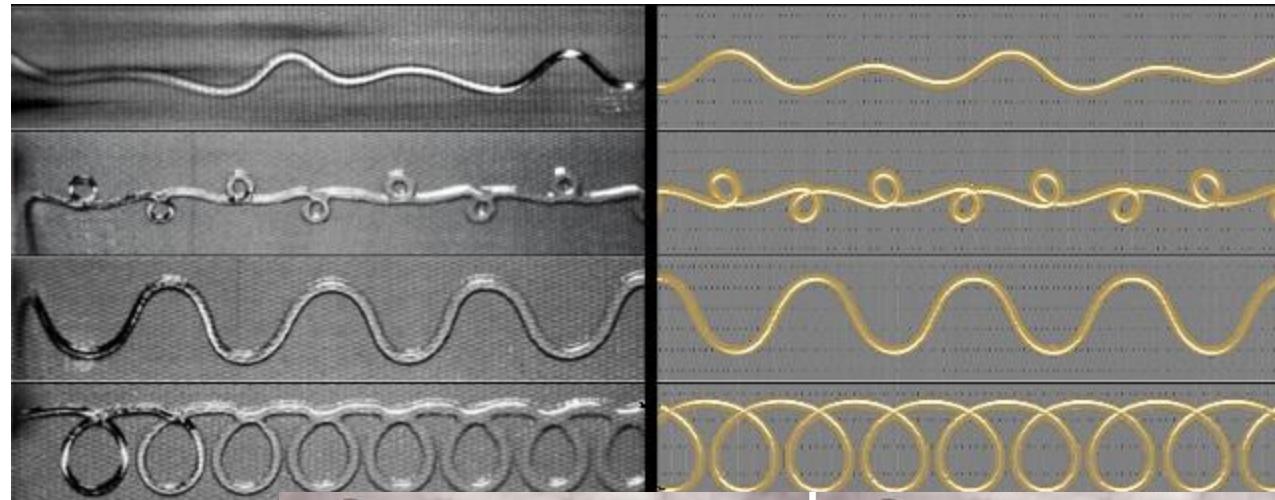
Shape collection analysis

Example Tasks

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Mapping

Example Tasks

Deformation transfer

Example Tasks

http://www.cs.columbia.edu/cg/threads/   http://mbergou.com/
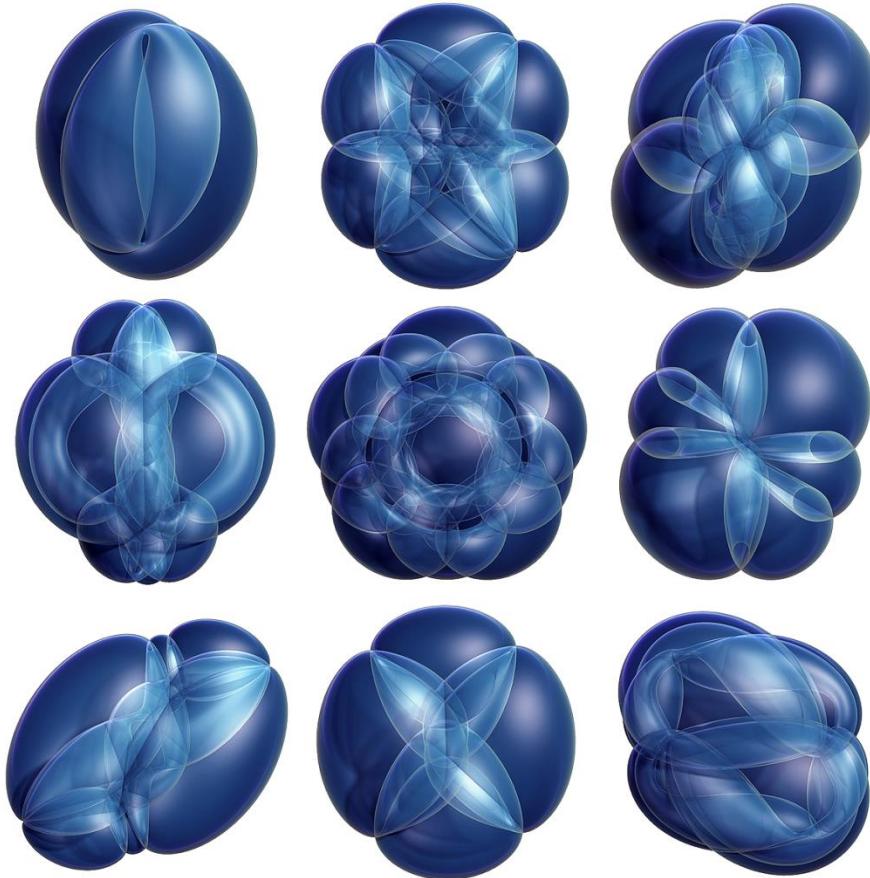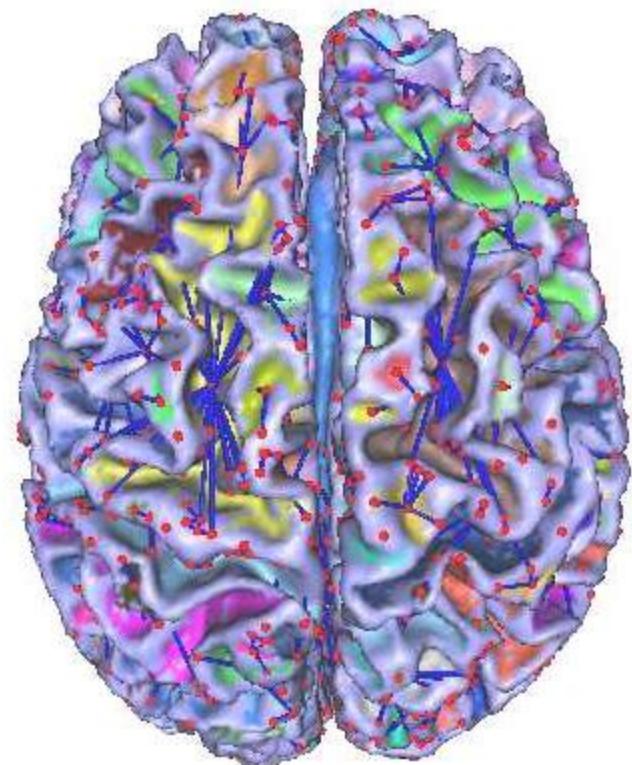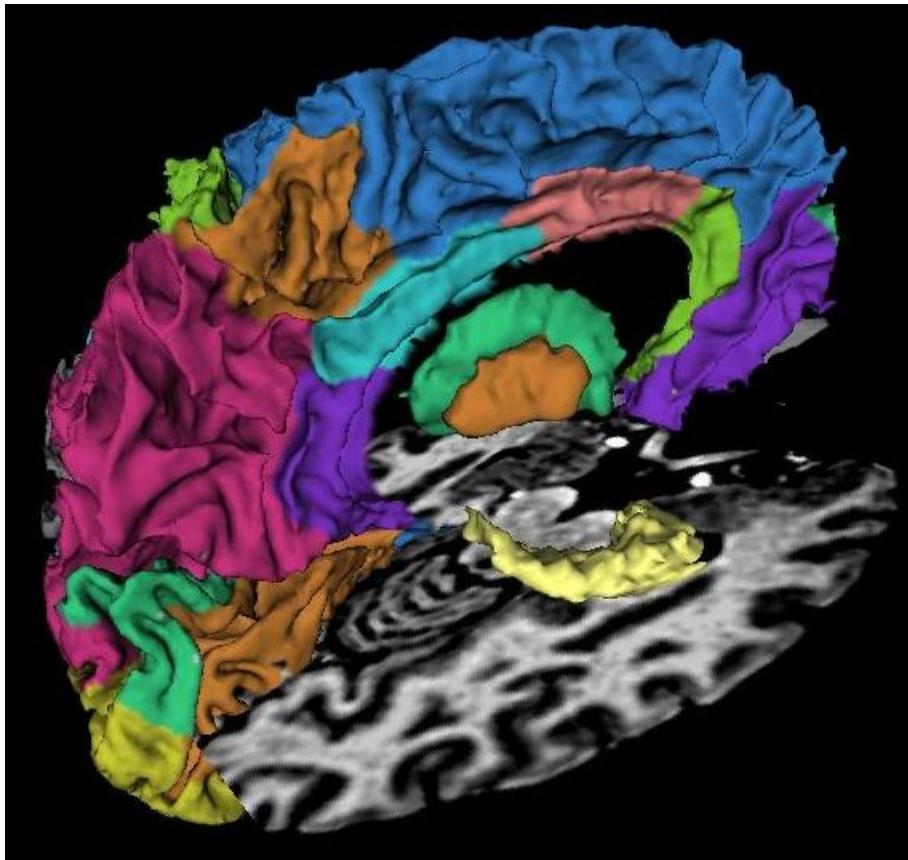
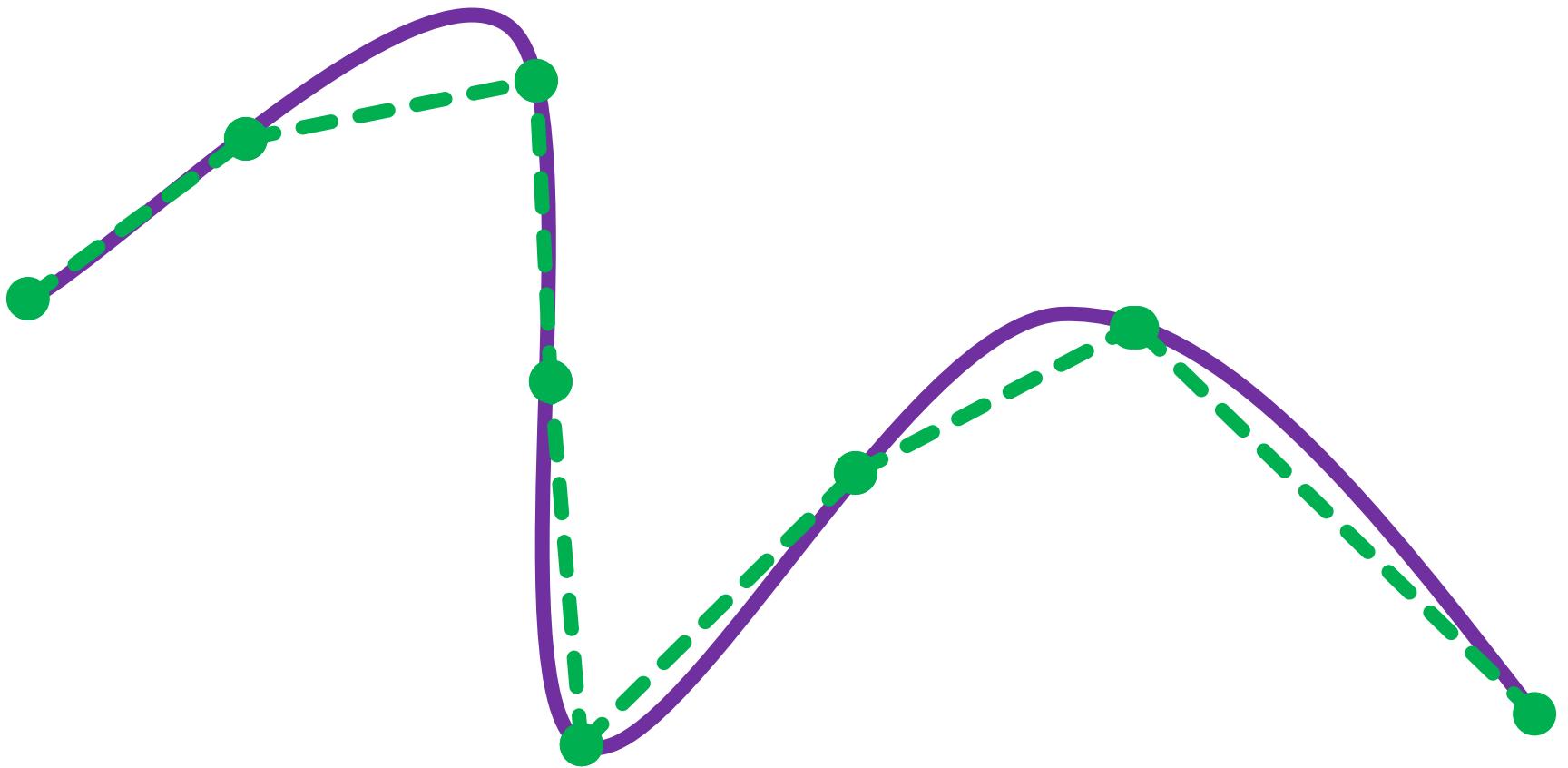## Simulation

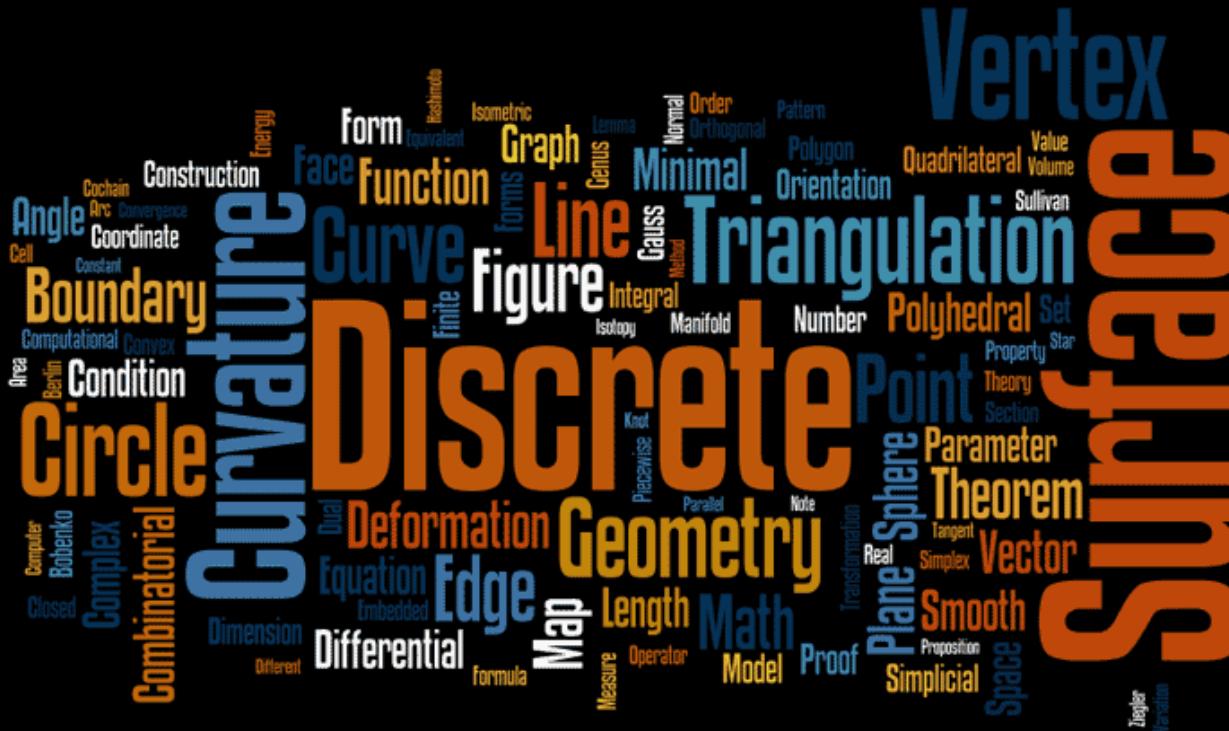# Example Tasks

Scientific visualization

Example Tasks

http://www.bioinformaticslaboratory.nl/twiki/pub/EBioScience/News/freesurfer-3d.jpg
http://hal.inria.fr/docs/00/40/21/30/IMG/vivodtzev_et_al-Dagstuhl03.jpg

Segmentation

What's Next?

Differential geometry of curves

Differential Geometry for Computer Science

CS 468, Spring 2013
Differential Geometry for Computer Science

Justin Solomon and Adrian Butscher