Elastically Deformable Models

A critique of the paper by Terzopoulos et al.

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1 citation

Elastically Deformable Models, D. Terzopoulos, J. Platt, A. Barr, and K. Fleischer. Proc. Siggraph 1987, pp. 205-214.

2 abstract

The paper uses elasticity theory to model the differential equations that govern the motion of elastic objects. The use of the physical model of forces acting on the objects allows a unified model for describing the shape and movement of the objects.

3 Summary

The paper presents a method of building a physical model for computer graphics. Traditionally, when building a model for computer graphics one specified the objects by shapes that were either stationary or moved according to trajectories specified by the model builder. The motivation of the paper is quite straight forward: since the model is of the physical world ant the physical world is governed by the laws of physics (that are known to us), we can build a model that acts according to these laws. This allows an active model in which forces and constraints are applied to the objects. Moreover, the objects can interact with one another. The paper deals specifically with deformable objects, based on simplification of elasticity theory. The use of the theory developed allows animation of complex objects acting under specified forces.

The model is of deformable objects. By Newtonian mechanics, the externally applied forces are balanced by forces due to the deformable model.

We consider an object Ω - either a solid body, a sheet or a curve. We denote by a the intrinsic coordinates inside Ω. We are interested in $r(a,t)$ for all points $a \in \Omega$ - the coordinates in the 3 dimensional space as they vary over time. Mathematically, the equations governing the motion are

$$\frac{\partial}{\partial t}(\mu \frac{\partial r}{\partial t}) + \gamma \frac{\partial r}{\partial t} + \frac{\delta E(r)}{\delta r} = f(r,t)$$

$\mu(a)$ is the mass density of the object, $\gamma(a)$ is the damping density, E is the functional that measures the potential energy due to the elastic deformation of the body. The balance is between the inertial force (the derivative of P), the damping force due to dissipation and the force due to the potential energy of the body's deformation (the derivative of the potential) on the one hand and the net external forces ($f(r,t)$) on the other.

Quite a lot of energy is devoted in the paper to the formulation of the potential energy of deformation $E(r)$ as derived from elasticity theory. The analysis is slightly different for the three cases - a solid body, a sheet and a curve. Solid bodies are uniquely described (up to rigid body motion) by their metric tensor, a matrix denoted by G. Sheets are described by their metric tensor and a curvature tensor B. Curves are described by their arc length, curvature and a torsion. The potential elastic energy is calculated using these measures in original undeformed body and the deformed body. For example, for a curve $E(r) = \int_{\Omega} \alpha(s - s^0)^2 + \beta(\kappa - \kappa^0)^2 + \gamma(\tau - \tau^0)^2 da$ where α, β, γ are the amount of resistance of the curve to stretching bending and twisting respectively. There are analog expressions for sheets and solid bodies (with matrices as parameters rather than just s, κ, τ).

The forces mentioned as external forces are gravity, fluids and collisions with impenetrable objects. The formulation of most of the external forces is quite straightforward - gravity acts by $f_{gravity} = \mu(a)G$, springs act by $f_{spring} = k(r_0 - r(a_0))$ etc. Collision dynamics between elastic objects and immobile impenetrable objects are modeled using a potential energy at the objects surface. The potential energy is chosen to be an exponentially increasing potential such that the energy is prohibitive if the model attempts to penetrate the object. Self intersection is avoided in the same manner - the object is surrounded by a self repulsive collision force. This requires an implicit description of the surface of the object. The authors are aware of the inefficiency of this part of the model, and even hint at a possible solution using hierarchical bounding boxes.

To create animation with deformable models the authors suggest solving the differential equations of motion numerically. The model has to be discretisized first - thus transforming the partial differentiable equation of motion mentioned above into a system of linked ordinary differential equations of the form $M\frac{\partial^2\vec{r}}{\partial t^2}+C\frac{\partial\vec{r}}{\partial t}+K(\vec{r})\vec{r}=\vec{f}$. The K is a matrix describing the stiffness, $\vec{f}$ is the the net external force, M is the mass matrix and C is the damping matrix. Then, the ordinary differential equations are integrated through time using a numerical step-by-step procedure by some small time intervals Δt. This transforms the differential equations into a sequence of linear algebraic systems that we can solve.

The authors simulated the behavior of several deformable objects to demonstrate the model's capabilities. Among others - simulations of an elastic surface lifted by a spring at one corner, of a ball on a deformable solid cube and of a flag waving in the wind are mentioned. The forces participating in the simulations include gravity, friction and viscous force. The pictures look as if the model is quite accurate.

4 Summary

It seems that the main problem of the described model is its efficiency. For instance, the collision detection mechanism although very elegant seems to be extremely inefficient. Efficiency aspects seem to be ignored by the authors altogether: the weak points are not mentioned and the descriptions of the simulations do not include any data regarding the amount of computation needed (other papers state length of computation in time units). Almost no complexity analysis is done either (of space or time). The upside of this is that as the paper is quite old (87') it is not clear that time measures would have had any meaning.

All in all it is a very nice paper. The motivation is elegant, the paper is clear, not tedious and yet does provide all the details needed to understand the model. There is some Newtonian physics and some PDE jargon needed to follow the text. However, it is written in a manner that bridges over the knowledge gaps for those who are not fluent in the physical aspects of the model. It seems to be a somewhat tricky task - presenting a model that is a simplification of a physical system to the CS community, and it is handled successfully.