Critique for CS448
Collision Detection

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Citation for Paper

E. G. Gilbert, D. W. Johnson, and S. S. Keerthi. A Fast Procedure for Computing the Distance Between Complex Objects in Three-Dimensional Space, *IEEE Journal of Robotics and Automation*, 4, 1988, pp. 193-203.

Abstract

The paper presents an efficient algorithm for computing the Euclidean distance between a pair of convex objects. The computational cost is approximately linear in the number of vertices specifying the two polytopes.

Summary

Problem Definition
Input: two finite sets of vertices whose convex hulls define polytopes
Output: the point in each of the polytopes that constitute the nearest points between the sets.

The goal of computing the distance between complex objects is to aid in solving collision detection and path planning problems. The fields of computer graphics, robotics, and computer-aided design require a highly efficient method to measure the distance of the nearest features of two objects. Collision detection enables animations and simulations to have a more realistic appearance and behavior.

The algorithm developed by Gilbert, Johnson, and Keerthi (GJK) in the paper has its origins in mathematical programming and uses a descent procedure on the distance between elementary polytopes in the convex sets. The general theoretical algorithm is to generate a sequence of convex polytopes whose vertices are a subset of the original vertices such that the near points of the convex polytopes to a given origin converge to the solution. As stated by Mirtich, the distance between the origin and the nearest simplex of the difference polyhedron is equal to the distance between the original polyhedra. When the convex polytopes collide, the degree of penetration is returned by the algorithm.

A subalgorithm for evaluating the distance between elementary polytopes contributes to the overall efficiency of the algorithm. It is stated that the algorithm terminates after a finite number of steps when the input is a compact and convex polytope. However, a backup procedure may be invoked to bring the algorithm to completion because numerical errors can cause the subalgorithm to fail. A number of steps can be taken to reduce errors in the algorithm and lead to a more efficient implementation. First, the origin of the system can be moved to be on the line joining the centroids of the vertex sets. Also, a special initialization based on knowledge of one state can be used instead of the single point initialization. The set most likely to produce the near point, based on the previous step, can be put at the beginning of the list.

The algorithm was tested on examples by selecting pairs of polytopes from a family of twelve. The number of vertices ranged from 2 to 100, so the geometries were not extremely complex. The examples studied the cases where the pairs of polytopes were separated, just touching and intersecting. A given pair of polytopes was perturbed with random translations and rotations to generate a large sample size. The algorithm ran to completion in all the cases, without requiring the backup procedure or failing to meet the error tolerance. The CPU time for running the algorithm was on the order of 10 to 30 ms. In general the situation where the objects were just touching were the most difficult based on number of flops per vertex of the objects.

The algorithm was also used on a specific example solved by Canny to compare the solution times. The problem involved finding the near points between an object and its environment as the object moved through time. The computational times for the GJK algorithm were significantly improved when a special initialization was used. As the number of grid intervals increased, the improvement seen by using the special initialization was greater.

The algorithm is designed for convex polytopes, but can be extended to nonconvex shapes by decomposing them into convex polytopes and their spherical extensions. A conservative approach would be to find the distance between the convex hulls of the nonconvex polytopes.

Comments

Though the algorithm developed by Gilbert, Johnson, and Keerthi has been modified and improved upon since the original presentation in 1988, the foundation of the method has served as the basis for some current algorithms. Q-Collide by Rabbitz uses the GJK algorithm for low-level collision detection, while Cameron has implemented the fastest descendent of GJK by enhancing the algorithm (Mirtich). The fact that the basic algorithm can be expanded to accommodate complex, nonconvex objects confirms the general applicability of the algorithm. The numerical experiments on simple polytopes showed that the algorithm performed well in practice.

The authors assume that the reader can readily understand the mathematical concepts utilized in the theorems and proofs of the paper. I had difficulty following the details behind the support properties of a given convex hull. The references to the Caratheodory theorem and Minkowski set difference also were not straightforward.

I would have liked to see a more complex topology presented to the algorithm to see if the CPU time increased linearly with the number of vertices. Because computational power has increased tremendously over the past decade, the availability of adequate processing speed at the time the paper was written may have been a problem. An example of decomposing a complex nonconvex object into the union of convex objects would have helped to demonstrate the robustness of the algorithm. This extension is mentioned in the paper but not implemented.

Discussion

It seems that when a complex object is composed of several simple convex polytopes, that the algorithm will need to search through more convex hull subsets, will the convergence still be linear?

The collision detection example is based on knowing the path of the object as a function of position and orientation. How could the algorithm adapt when the path of the object is uncertain?

Reference

Mirtich, Brian. V-Clip: Fast and Robust Polyhedral Collision Detection, Mitsubishi Electric Research Laboratory, Technical Report TR-97-05 (1997).