Critique for CS448B: Modeling in Motion

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1. Citation for Paper

Tamal K. Dey, Herbert Edelsbrunner, Sumanta Guha and Dmitry V. Nekhayev, **Topology Preserving Edge Contraction.** Publications de l'Institut Mathématique (Beograd), Vol. 60 (80), 1999. Also, Technical Report RGI-Tech-98-018, Raindrop Geomagic Inc., Research Triangle Park, North Carolina, 1998.

2. Synopsis

This paper focuses on edge contractions in simplicial complexes, which preserve topological type. It gives out a link condition and proves its correctness for 2-complex and 3-complex.

3. Summary

Edge contraction is useful in simplifying a triangulated surface for fast rendering. But topological property should be preserved in edge contraction. This paper gives out a local condition called **Link Condition** for edge contraction preserving topology. It proves that for 2-complexes the edge contraction satisfying link conditions has a local unfolding. For 2-manifolds, all edge contractions that have an unfolding also have a local unfolding. So link condition characterizes all the edge contractions that preserve topological type. This result can be extended to dimension 3. The only change is that for 3-complexes, the edge contraction satisfying link condition has a relaxed unfolding.

This paper first gives out the definition of combinatorial equivalence between two simplicail complexes. And then it defines topology preserving edge contraction. An edge contraction ab makes a simplicial complex K to be a new simplicial complex L by replacing $\overline{St}(ab)$ with $St(c)$. If there is a homoeomorphism f from underlying space of K to underlying space of L that differs from the identity only inside the star of ab, then f is called a local unfolding. Each unfolding corresponds to a pair of isomorphic subdivision of K and L. So if there is a local unfolding, that means K and L are combinatorially equivalent. So to prove that an edge contraction preserves topology turns out to prove that there is a local unfolding between K and L.

Secondly, the authors bring out the Link Condition: $Lka \cap Lkb = Lkab$. The paper gives out a more complex and tighter definition. But the idea is actually the same. The authors then show us a tight proof for the equivalence of link condition and the property of topology preserving in a constructive way. The main idea is to construct subdivisions for K and L by adding some vertices and edges, and then prove the subdivision of K is isomorphic to the subdivision of L. By the definition, we know K and L are combinatorially equivalent.

At last, the authors bring up a couple of open questions in this area.

4. Comments

Edge contraction is an important way to make the surfaces simpler. But it must preserve the topological type of the surfaces. In computer graphics, we are interested in a surface consists of triangles in R^3 which connects to each other along edges and vertices. Such surface is a 2-complex in mathematics. So study of edge contraction for 2-complex is sufficient for practical use. The main contribution of this paper is that it gives out the sufficient and necessary condition for an edge contraction to preserve topology for 2-complexes and 3-complexes and its tight proof. Additionally the link condition is intuitive and gives us a deeper understanding of the properties of edge contraction. The advantage of link condition is that it is a local condition, which only involves of a small number of nearby vertices. That means the algorithm can only inspect the nearby area instead of searching all the vertices.

This paper discussed edge contraction from a mathematical point of view. The authors use a lot of time to introduce definitions, and use several pages to prove the theorems. It doesn't give out a concrete algorithm for how to verify the link condition, but it do give out a structural proof which shot a light on how to find a homoeomorphism between the underspace of the original simplicial complex and the changed one.

On the other hand, topological equivalence is not the most important criteria that we should consider in edge contraction. If the edge is so small that people will not notice it, it doesn't matter that we replace the edge with a vertex even if it doesn't preserve topological type. For a long edge, although we know the edge contraction will preserve topology, shouldn't be contracted cause the resulting complex will appear quite different with the original one.

5. Questions for discussion

(1). Is this approach of proving an edge contraction preserves topology fast enough for implementation? What is the cost to prove or disprove that an edge contraction preserves topology? Will it need any particular data structure to represent the complex?
(2). If we have two legal edge contractions, which one is better? What are the criteria? Will an edge contraction affect the following edge contractions? If yes, how much is it?