# Critique for CS448B

# ARTDEFO: Accurate Real Time Deformable Objects

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## Citation for Paper

Doug L. James and Dinesh K. Pai, **ARTDEFO: Accurate Real Time Deformable Objects**, Proceeding of *SIGGRAPH*, pp.65-72, 1999.

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## Synopsis

The paper presents an algorithm for physically accurate simulation of elastic deformable objects fast enough for real-time animation and interaction.

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## Abstract of Paper

The paper presents an algorithm for physically accurate simulation of elastic deformable objects that are also fast enough for real-time animation. The algorithm uses Boundary Element Method (BEM) (vs. Finite Element Method). The algorithm applies to deformable objects that can be simulated by linear elastic models. It achieves interactive speeds by exploiting coherence in typical interactions since BEM lends itself to fast updates when few boundary conditions change. The methods is presented in mathematical detail along with examples.

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## Summary

Simulating deformable objects for graphics is difficult because of the conflicting demands on performance and accuracy. Performance is important for real time feedback and interaction while physical accuracy enable ease of modeling and usage. Traditional deformable modeling can be broadly classified into: interactive methods like spline based or mass-spring models that gives interactive performance but requires lots of user control and tweaking; physics based models which start with the laws of physics and solves the PDE's numerically using FEM. Physics based models are accurate but are typically simulated off-line. This paper bridges the gap between interactive performance and physical accuracy for a certain type of deformable objects – linear elastic objects. The algorithms has two main innovations: 1) the use of boundary element method (BEM). The advantage of BEM over FEM lies in the reduction of the unknown. Since most of the deformation of interest to graphics applications lie on the boundary (displacement at the boundary, traction forces), which matches exactly the unknowns in BEM, while FEM also solves for unknown in the interior. Not solving the unknown in the interior also means this method can not be used to model large deformations (for example, deformation in human tissue). 2) the use of Sherman-Morrion-Woodbury formula to exploit coherence among linear deformations, linear models allow many system responses (deformations) to be precomputed and combined in real-time using low-rank update construction when the amount of update to the boundary is small as in typical interactions. Again this condition limits the applicable animations that can be generated in real-time using this system, even small deformation that covers a large area can a pose a problem (because it will cause many changes to the boundary). "Touch but don't squeeze"!

### *Mathematical Formulations*

The paper gave a clear but brief exposition on the mathematical details in this algorithm. The following concepts are presented in order:

1. Navier's Equation of Linear Elasticity
2. Boundary Value Problem defined by the boundary condition of Navier's equation
3. Boundary Integral Formulation (analytical solution to BVP)

### *Boundary Element Method*

Boundary element method is the numerical discretization of the boundary intergral formulation, it consist of three main steps:

1. Discretize the boundary into N non-overlapping elements which represents the displacements and traction forces by functions which are piecewise interpolated between the elements' nodal points.
2. Apply integral equations at each of the n boundary nodes, perform resulting intergrals over each boundary element in order to generate an undetermined system of 3n equations involving 3n nodel displacements and 3n nodal traction forces.
3. Apply the boundary conditions of the desired boundary value problem, fixing n nodel values per direction, the remaining 3n linear system may be solved to obtain unknown nodal boundary values.

Note as in FEM, step 1 and 2 are preprocessing steps because the topology of the discretization is not changed by the deformation (i.e. no fractures).

### *Interactive Performance*

Even though the final step of BEM involves a smaller linear system than the large sparse one as in FEM, real-time solution to BEM is still not trivial. The linear system is dense and it is too large to be solved in real time afresh, yet boundary condition changes (caused by human interaction with the object for example) will cause the linear system to change. To achieve real time performance, the algorithm precomputes a set of linear systems corresponding to a set of reference boundary value problems (condtions). If the changes to the boundary condition only cause a few columns to change in the linear system, the algorithm uses the Sherman-Morrison-Woodbury formula to reconstruct the new inverse matrix using the precomputed linear systems.

# General Comments

The paper is well written and documented. The authors state their problem domain clearly and show an innovative solution to that set of problems. The mathematical and physics formulations that form the basis of their method are presented clearly without going into too much detail. It's a bonus that they implemented the system in Java and provide interactive demos in real-time. http://www.cs.ubc.ca/~djames/deformable

The paper introduces the usage of Boundary Element Method to deformable modeling in 3D graphics. It presents a fast and accurate algorithm to simulate small deformations in linearly elastic objects. In general, the main advantage of BEM over FEM is the reduction of unknown. BEM solves for forces and displacement on the boundary while FEM solves for forces and displacement over the entire object. Since most visual applications tend to be more interested in the boundary, BEM seems promising. On the other hand forces and displacement in the interior definitely affect the boundary, ignoring stress and internal deformations will reduce the physical accuracy of the simulation. For applications like virtual surgery where interior deformation is necessary, FEM is still the right solution.

Another key idea of the paper is to use of low rank update reconstruction to achieve real time simulation of small deformations. This type of reconstruction is not limited to BEM, a similar technique exists in FEM as well, called "structural reanalysis", one wonders whether or not that is an area worth looking into for 3D deformable modeling?

The paper also claims the fact that BEM can share the same discretization (meshing) as the one used to rendering as another advantage over FEM (which requires 3D meshing). Often the meshing requirements for rendering and numerical methods are quite different and sometimes contradictory, for complicated boundary surfaces, how realistic is it to use the same discretization for both?

# Issues & Questions

In this last section of the course, we have seen many different approaches to deformable modeling. The trade off between performance and quality is ever present. Because of the domain of graphics and animation, often the quality stresses on visual accuracy rather than physical accuracy. Not because physical accuracy is not desired, visual accuracy is chosen due to performance and cost constraint. We are trying to spend our money on where it most counts – visual accuracy. The paper seems to offer the best of both worlds, physical accuracy (which implies visual accuracy) at interactive real-time performance. What is the magic? Is it really that Boundary Element Method is so superior to Finite Element Method, that we have all been using the wrong algorithm all along? No. The magic lies in the selection of the problem domain. Instead of trying to build a simulation system for general deformable objects that range from highly elastic to highly stiff, it targets deformable objects that can be simulated well using linear elastic models. Instead of allowing arbitrary interactions and constraints, the algorithm targets interactions that only changes a few boundary elements thus will only incur a small update cost to the boundary element model. The examples look interesting and demonstrate the power of this method well, but at the same time, they all consist of objects and interactions that are well supported by the algorithm; smooth round elastic objects have simple boundary, a gentle poke by a finger causes small changes to the boundary. It's not clear whether or not this algorithm can be extended to deal to highly deformable material like human tissue or highly stiff objects like cloth or even allow more complicated interactions? It's ironic the authors opened their paper with "so much of our physical world can not be modeled by rigid bodies, the most important examples are humans, human tissues, etc". I am sorry to say, "so much of our world can not be modeled by smooth round (simple boundary) Jellos either, nude humans don't resemble Jello, neither do the clothes ones." We have a long way to go in terms of modeling the world.

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