Critique for CS448A

Large Steps in Cloth Simulation

Citation for Paper

David Barraff, Andrew Witkin, **Large Steps in Cloth Simulation**, Proceedings of *SIGGRAPH*, 43-54, 1998.

Synopsis

This paper presents a cloth simulation system that can stably take large time steps resulting in faster running times. The stability results from the use of an implicit integration method to solve the ODE of motion combined with a new technique to enforce constraints.

Abstract of Paper

This paper presents a cloth simulation system that can stably take large time steps. The system combines a new technique for enforcing constraints on the cloth particles with an implicit integration method. The simulator discretizes the cloth as a triangular mesh. It uses a simple continuum formulation to derive the internal cloth forces that model operations like local anisotropic stretch or compression. It also includes a unified treatment of damping forces. A modified conjugate gradient method is used to solve the large sparse matrix generated at each time step of the implicit method. Due to the modification, the constraints are enforced exactly at each iteration step. The modified conjugate gradient method converges at a similar rate to unmodified CG.

Summary

Physically based cloth animation is of interest to computer animation and poses a significant challenge because of the computational and mathematical complexity. The main challenges in cloth animation are the following: numerical instability due to stiffness of the material, high resolution requirement to show the realistic wrinkling and folding of cloth, constraint maintenance between cloth and solids or between cloth and cloth.

The simulation system presented here provides a framework that meets all of the above challenges making it viable and useful for generating realistic animations of non-trivial cloth models.

Representation

Cloth is modeled as a triangular mesh of particles. Each particle has an associated position, velocity, and force acting on it. The simulator computes the position and velocity for each particle at each time step.

Implicit Integration Method

Physically based cloth simulation is formulated as a time varying PDE that after discretization is solved as an ODE:

The stability of this cloth simulation system results from the use of an implicit integration method rather than explicit method like Euler's method. Explicit methods require small time steps to maintain stability and typically perform poorly on stiff systems. In fact, the largest step that can be taken in explicit methods is less than (2/k) where k is the stiffness constant. For system with some very stiff components, the entire simulation can grind to a halt.

On the other hand the implicit method is based an implicit *backward Euler* method using a Taylor series expansion to convert the resulting non-linear system into a large unbanded sparse linear system. Without constraints, this linear system can be converted to a symmetric, positive definite system and solved using a conjugate gradient method with a running time of roughly $O(n^{1.5})$. While the running time for an explicit method, like Euler, is O(n), the implicit method permits the use of much larger time steps without compromising stability. This in turn allows higher resolution models to be simulated without increasing the overall running time.

Forces

Cloth's material behavior is often described in terms of a scalar potential energy E. For practical reasons, E is decomposed into set of sparse energy functions. In this paper, they specify a vector condition C(x) which should be zero when the system is at equilibrium. The corresponding energy is defined as $\frac{1}{2}kC(x)^T C(x)$, k is the stiffness constant. The quadratic energy formulation with arbitrarily large stiffness is chosen because it fits well with the numerical model of the simulation system (can deal with arbitrary stiffness). Simple and intuitive continuum-based force models are developed that allow the system to support operations like local anisotropic stretching, shrinking, and bending. Damping forces are easily derived using this energy formulation and integrated into the system.

Constraint Maintenance

The position and velocity of a particle can be constrained by attaching it to a fixed or moving point in space, or by limiting it to a fixed or moving curve or surface. Cloth/solid contacts result in automatically generated constraints in which a particle is attached to the surface, or in the case of a low-friction contact, constrained to remain on the surface with sliding allowed. To reduce the computational burden cloth/cloth contacts are modeled using penalty forces. A particle that is near or penetrating a cloth triangle is penalized with a stiff spring with damping to restore the particle back to the correct side of the triangle.

For explicit methods, direct constraints can be easily maintained by reducing the DOF of the particle. For the implicit system, reducing DOF of the particles means changes to the structure of the linear system to be solved at each time step; this would be too costly an option. Instead, cloth/solid contact and user supplied constraints are enforced using a *mass modification* method. The linear system generated by the implicit integrator uses the inverse mass of the particles in the system. Constraints are enforced by altering the inverse mass of a particle. By setting the inverse mass of a particle to zero, the mass becomes infinite and the particle's velocity becomes constant (the acceleration is zero). To constrain the acceleration of a particle in a particular direction the inverse mass can be written as a matrix rather than a scalar with the inverse mass contribution along the constraint direction set to zero. The method is generalized to allow a specified acceleration, rather than zero, in a specified direction by adding an extra term to the linear system representing the velocity change in the constraint direction.

Modified Conjugate Gradient Method

The linear system with mass modifications cannot be directly solved using the conjugate gradient method, because the modified mass matrix is singular so the system cannot be trivially converted to a symmetric system. Instead, the traditional CG method is modified to include a filter that guarantees the constraints exactly at each step of the iteration. This simple modification also has the side benefit of allowing the system to compute the force needed to release the constraint. The modified CG method converges at a rate of $O(n^{1.5})$ empirically.

Collision

The system doesn't introduce a new collision detection method, instead it does have a quick and robust way of handling penetration. Penetrations between cloth/cloth particles are handled using a penalty force. For cloth to solid penetration, the system makes an alteration to the position that is integrated into the ODE. In combination with mass alteration, the system has total control of the velocity and position of the constrained particle.

General Comments

The paper presented a cloth simulation system that shows simulation results that are comparable in quality with the current state of art, but computed an order of magnitude faster. It is definitely a large step forward in cloth simulation (as the title suitably claims).

The paper is well written. It gave a good introduction and motivation to the development and use of an implicit integration method in a cloth simulation system. It also presented all the key components of the system and the reason why the system chose a particular solution or model to deal with a specific challenge clearly. The empirical results are well documented and match the qualitative computation cost analysis and predictions made by the paper.

Issues and Questions

My main complaint with the paper is the computation cost analysis can be more rigorous, especially the cost comparison between the explicit method and implicit method (the main contribution of the paper). The force models developed in the paper are simple, intuitive, and provide good visual realism, but lack justification.

Given the great performance of the system and the fact that the results are measured on relatively old hardware, one wonders what can be accomplished on current processors. Can this technology be used to implement interactive dynamic systems? What size models can be simulated interactively?

Author of Critique

Feng Xie
Ph.D. student, Stanford Computer Science Department
E-Mail: feng@graphics.stanford.edu
