Radiosity

Classic radiosity = finite element method

Assumptions

- Diffuse reflectance
- Usually polygonal surfaces

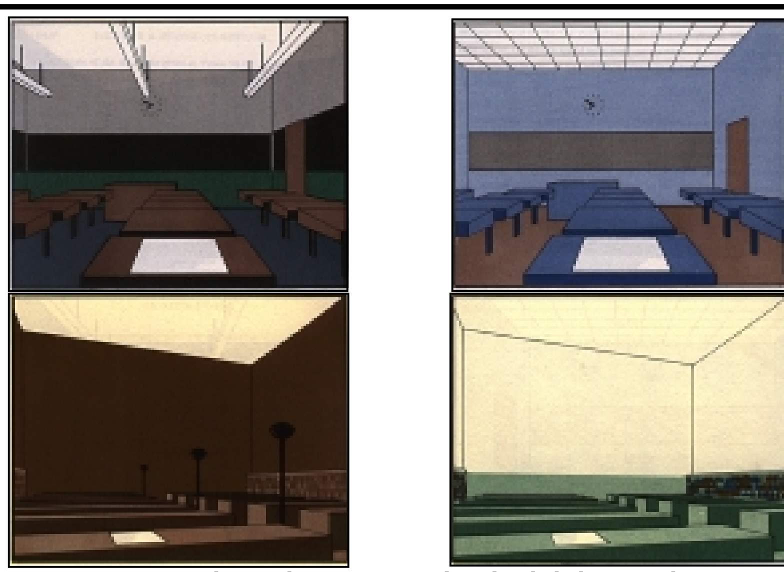
Advantages

- View independent solution

Techniques

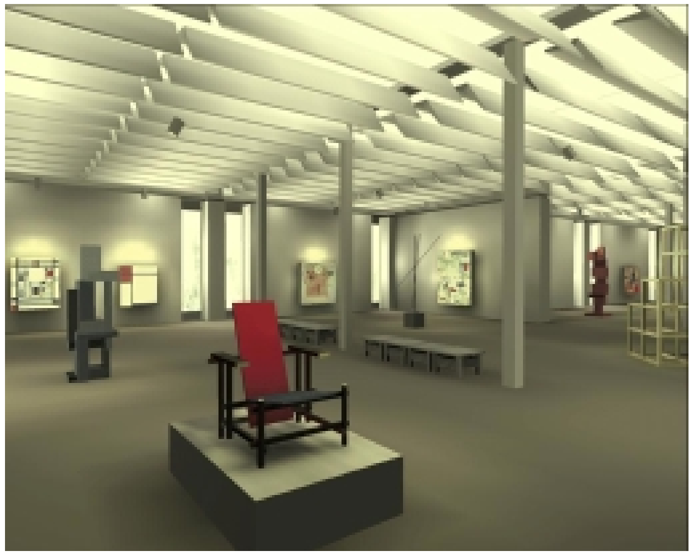
- Meshing
- Form factors
- Solving linear equations

First Radiosity Pictures …



Parry Moon and Domina Spencer (MIT), Lighting Design, 1948

Early Radiosity

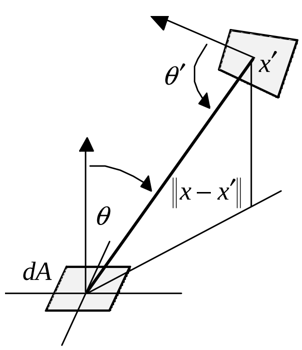


CS348B Lecture 17 Pat Hanrahan, Spring 2001

The Radiosity Equation

Assume diffuse reflection only

Solve for radiosity (2D function)



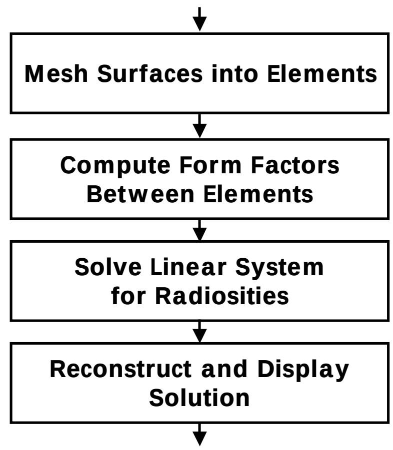
$$B(x) = B_e(x) + \rho(x) E(x)$$

$$B(x) = B_e(x) + \rho(x) \int_{M^2} F(x, x') B(x')\, dA'$$

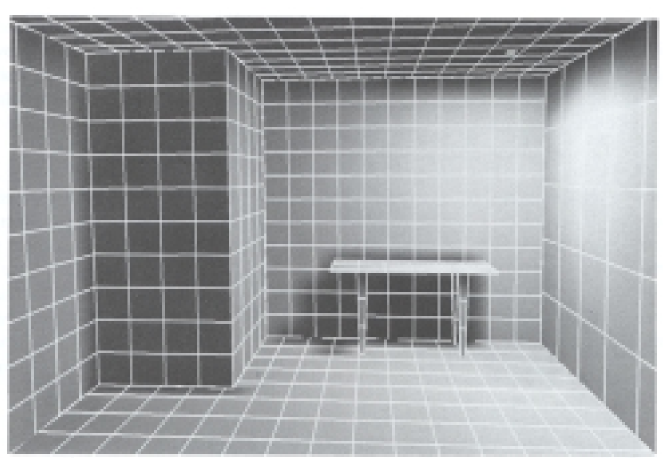
$$F(x, x') = \frac{G(x, x')}{\pi} = \frac{\cos\theta \cos\theta'}{\pi \left\| x - x' \right\|^2} V(x, x')$$

CS348B Lecture 17 Pat Hanrahan, Spring 2001

Classic Radiosity Algorithm



Simple Room Scene



Example from John Wallace

Derivation

Radiosity integral equation

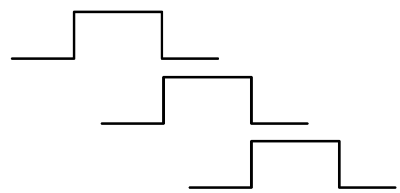
$$B(x) = B_e(x) + \rho(x) \int_{M^2} B(x')\, F(x,x')\, dA'$$

Piecewise constant basis functions

$$B(x) = \sum_i B_i N_i(x)$$

$$B_e(x) = \sum_i E_i N_i(x)$$

$$\rho(x) = \sum_i \rho_i N_i(x)$$



Derivation

Convert integral equation to matrix equation

$$B(x) = B_e(x) + \rho(x) \int_{M^2} F(x,x') \sum B_j N_j(x')\, dA'$$

$$\sum_i B_i N_i(x) = \sum_i B_i N_i(x) + \sum_i \rho_i N_i(x) \left[\sum_j B_j \int_{M^2} F(x,x') N_j(x')\, dA' \right]$$

$$\int \left(\sum_i B_i N_i(x) = \sum_i B_i N_i(x) + \sum_i \rho_i N_i(x) \left[\sum_j B_j \int_{M^2} F(x,x') N_j(x')\, dA' \right] \right) dA$$

$$B_i A_i = E_i A_i + \rho_i \sum_j B_j \int_{M^2} \int_{M^2} F(x,x') N_i(x) N_j(x')\, dA\, dA'$$

CS348B Lecture 17 Pat Hanrahan, Spring 2001

Form Factor

Throughput

$$T_{ij} = T_{ji} = \int_{A_i}\int_{A_j} \frac{\cos\theta_o' \cos\theta_i}{\pi \|x - x'\|^2} \; V(x, x') \; dA dA'$$

Reciprocity

$$T_{ij} \equiv A_i F_{ij}$$
$$T_{ji} \equiv A_j F_{ji}$$
$$\Rightarrow \quad A_i F_{ij} = A_j F_{ji}$$

Summation

$$\sum_j F_{ij} = \sum_i F_{ji} = 1$$

Form factor is the percentage of light ...

Classic Radiosity

Power Balance

$$B_i A_i = E_i A_i + \rho_i \sum_j B_j A_j F_{ji}$$

Reciprocity

$$A_i F_{ij} = A_j F_{ji} \Rightarrow B_i = E_i + \rho_i \sum_j F_{ij} B_j$$

Radiosity System

$$\begin{pmatrix} 1-\rho_1 F_{11} & -\rho_1 F_{12} & \cdots & -\rho_1 F_{1n} \\ -\rho_2 F_{21} & 1-\rho_2 F_{22} & \cdots & -\rho_2 F_{21} \\ \vdots & \vdots & \ddots & \vdots \\ -\rho_n F_{n1} & -\rho_n F_{n2} & \cdots & 1-\rho_n F_{nn} \end{pmatrix} \begin{pmatrix} B_1 \\ B_2 \\ \vdots \\ B_n \end{pmatrix} = \begin{pmatrix} E_1 \\ E_2 \\ \vdots \\ E_n \end{pmatrix}$$

Form Factors

Differential-differential - *F(x,x') dA'*

$$F_{dA_i,dA_j} = \frac{\cos\theta_o' \cos\theta_i}{\pi \|x - x'\|^2} V(x,x')\, dA'$$

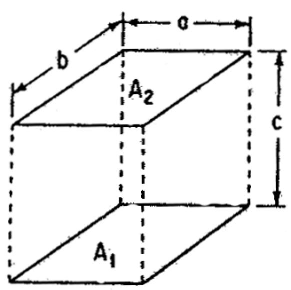
Differential-finite – *$E_j(x)$*

$$F_{dA_i,A_j} = \int_{A_j} \frac{\cos\theta_o' \cos\theta_i}{\pi \|x - x'\|^2} V(x,x')\, dA'$$

Finite-finite – $\Phi_{i,j}$

$$F_{A_i,A_j} = \frac{1}{A_i} \int_{A_i} \int_{A_j} \frac{\cos\theta_o' \cos\theta_i}{\pi \|x - x'\|^2} V(x,x')\, dA\, dA'$$

Analytical Form Factors



$$X = \frac{a}{c}$$

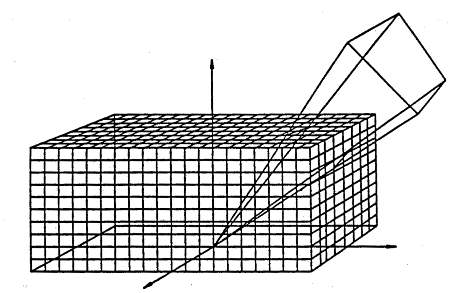
$$Y = \frac{b}{c}$$

$$F_{A_1,A_2} = \frac{2}{\pi XY} \left\{ \begin{array}{l} \ln\left[\frac{(1+X^2)(1+Y^2)}{(1+X^2+Y^2)} \right]^{1/2} + X\sqrt{1+Y^2} \tan^{-1} \frac{X}{\sqrt{1+Y^2}} \\ +Y\sqrt{1+X^2} \tan^{-1} \frac{Y}{\sqrt{1+X^2}} - X \tan^{-1} X - Y \tan^{-1} Y \end{array} \right\}$$

Hemicube Algorithm

First radiosity algorithm to deal with occlusion

1. **Render scene from the point of view of each vertex/element**
2. **Compute delta form factors – contribution from each pixel**



Render source elements from POV of receiving element

↓

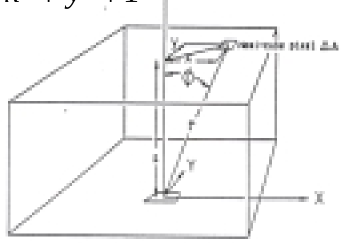
$$F_{dA_i,A_j} = \sum_{p \in A_j} \Delta F_p$$

↓

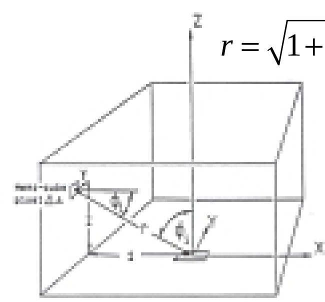
Typical resolution: 32x32

Hemicube Delta Form Factors

$$r = \sqrt{x^2 + y^2 + 1}$$



$$r = \sqrt{1 + y^2 + z^2}$$



$$\cos\phi = \frac{1}{\sqrt{x^2 + y^2 + 1}}$$

$$\cos\phi = \frac{1}{\sqrt{1 + y^2 + z^2}}$$

$$\Delta F = \frac{\Delta A}{\pi(x^2 + y^2 + 1)^2}$$

$$\Delta F = \frac{\Delta A}{\pi(1 + y^2 + z^2)^2}$$

Hemicube Algorithms

Advantages

- + First practical method -> Patent!
- + Use existing rendering systems; Hardware!
- + Computes all form factors in $O(n)$

Disadvantages

- - Computes differential-finite form factor
- - Aliasing errors due to sampling
  - Randomly rotate/shear hemicube
- - Proximity errors
- - Visibility errors
- - Expensive to compute a single form factor

Solve [F][B] = [E]

Direct methods: $O(n^3)$

- Gaussian elimination
  - Goral, Torrance, Greenberg, Battaile, 1984

Iterative methods: $O(n^2)$

Convergence

Energy conservation -> diagonally dominant -> converge

- Gauss-Seidel, Jacobi: <u>Gathering</u>
  - Nishita, Nakamae, 1985
  - Cohen, Greenberg, 1985
- Southwell: <u>Shooting</u>
  - Cohen, Chen, Wallace, Greenberg, 1988

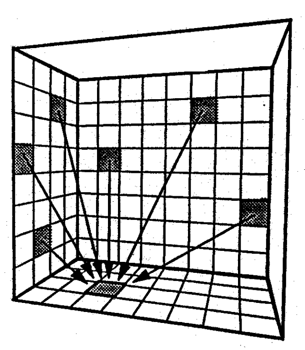
Iterative Solvers

Iteration

$$(I - F)^{-1} B = E$$
$$B = (I + B + B^2 + \cdots)E$$

$$B^0 = E$$
$$B^1 = E + FB^0$$
$$\dots$$
$$B^n = E + FB^{n-1}$$

Gathering



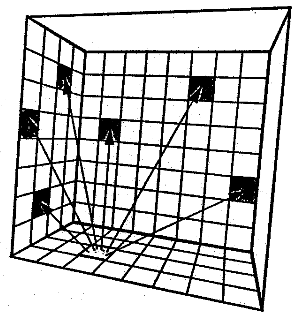
```
for(i=0; i<n; i++)
   B[i] = Be[i];

while( !converged ) {
   for(i=0; i<n; i++) {
      E[i] = 0;
      for(j=0; j<n; j++)
         E[i] += F[i][j] * B[j];
      B[i] = rho[i]*E[i];
   }
}
```

Row of F times B

Calculate one row of F and discard

Shooting



```
for(i=0; i<n; i++)
  B[i] = dB[i] = Be[i];
while( !converged ) {
  set i st dB[i] is the largest;
  for(j=0;j<n;j++)
    if(i!=j) {
      dBj = rho[j]*F[j][i]*dB[i];
      dB[j] += dBj;
      B[j] += dBj;
    }
  dB[i]=0;
}
```

Brightness order

Column of F times B

Results: Gathering vs. Shooting

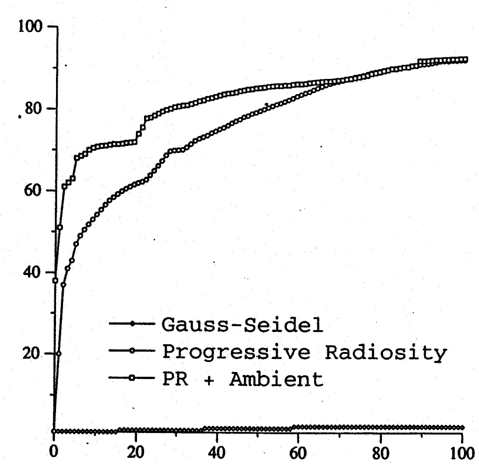
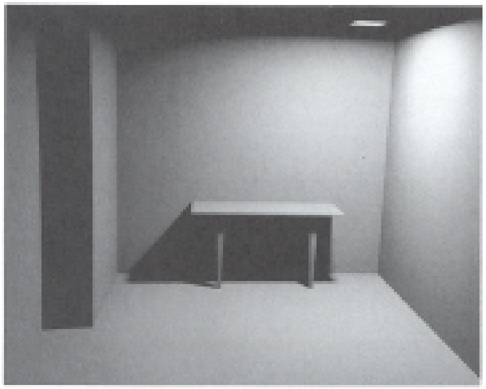
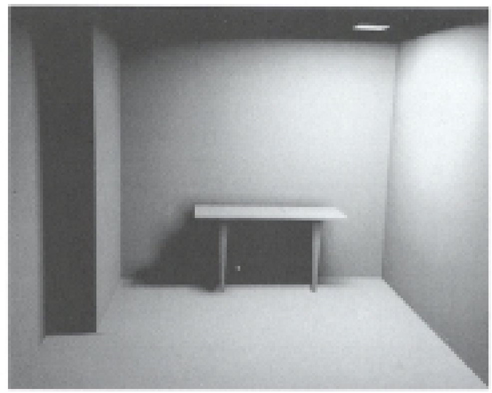


Figure 5.9: *Convergence versus number of steps for three algorithms.*

From Cohen et al.

Accuracy



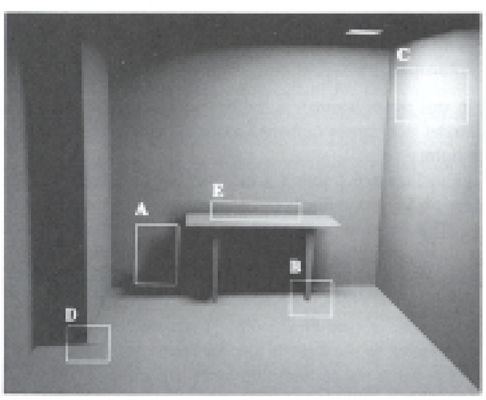


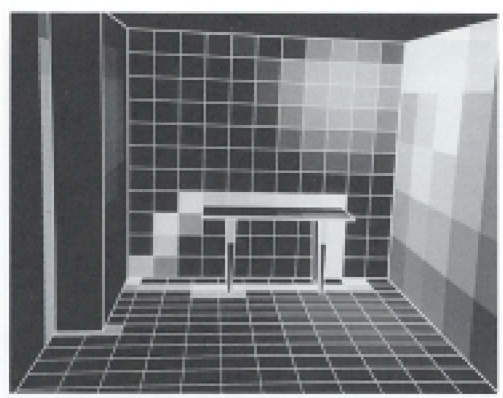
Reference Solution

Uniform Mesh

Table in room sequence from Cohen and Wallace

Artifacts

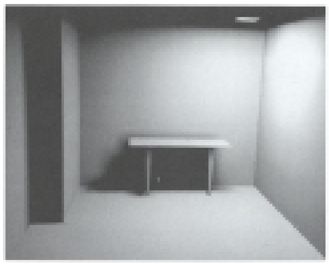


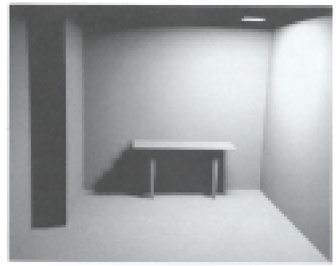


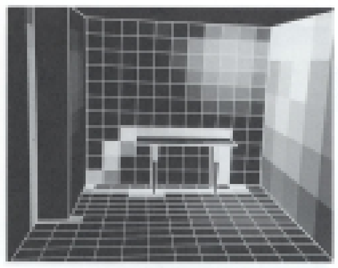
A. Blocky shadows
B. Missing features
C. Mach bands
D. Inappropriate shading discontinuities
E. Unresolved discontinuities

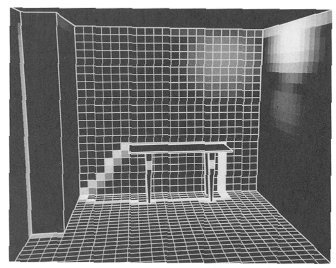
Error Image

Increasing Resolution



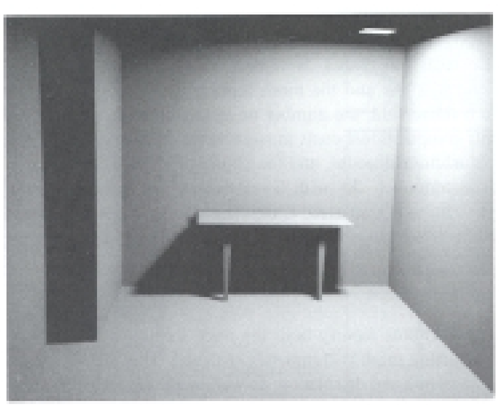


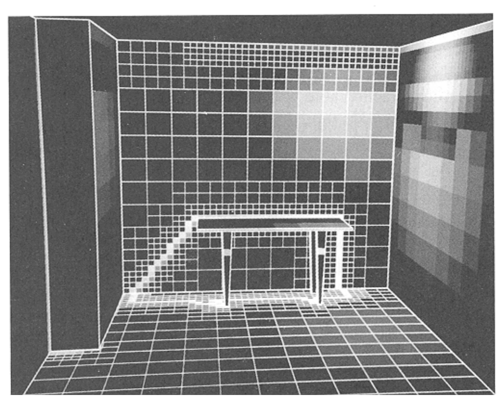




CS348B Lecture 17 Pat Hanrahan, Spring 2001

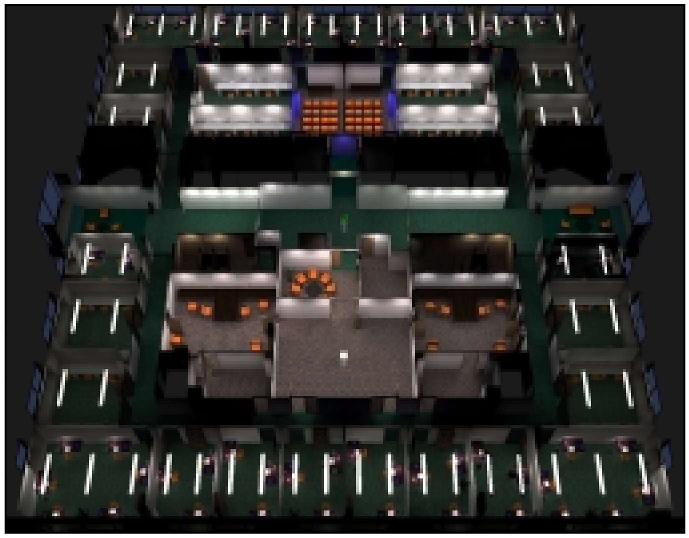
Adaptive Meshing





CS348B Lecture 17 Pat Hanrahan, Spring 2001

Megaradiosity



From S. Teller, T. Funkhouser, C. Fowler, P. Hanrahan