Ray Tracing

Today

- Basic algorithm
- Ray-Surface intersection for single surface

Next lecture

- Acceleration techniques for large numbers of objects

Classic Ray Tracing

Greeks: *Do light rays proceed from the eye to the light, or from the light to the eye?*

Gauss: Rays through lenses

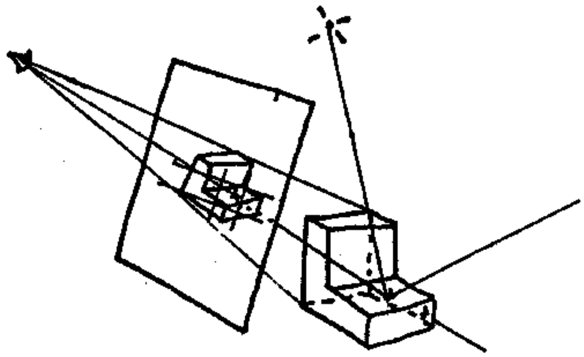
Three ideas about light

1. Light rays travel in straight lines
2. Light rays do not interfere with each other if they cross
3. Light rays travel from the light sources to the eye, but the physics is invariant under path reversal (*reciprocity*).

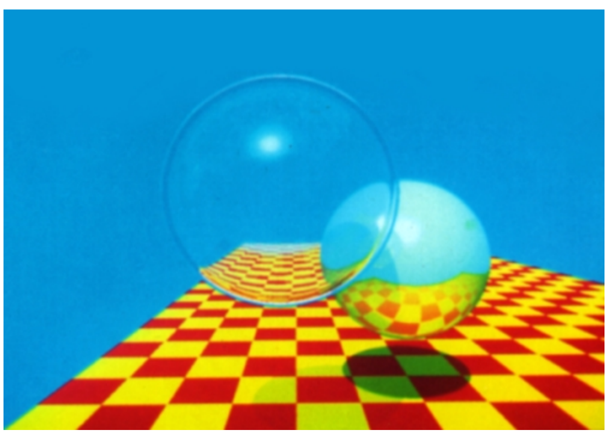
Ray Tracing in Computer Graphics

Appel 1968 - Ray casting

1. Generate an image by sending one ray per pixel
2. Check for shadows by sending a ray to the light



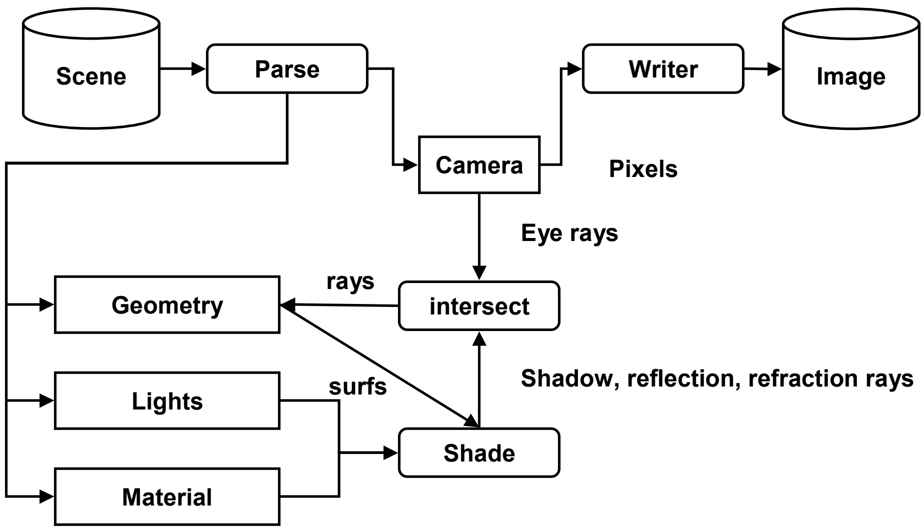
Ray Tracing in Computer Graphics



Whitted 1979

Recursive ray tracing (reflection and refraction)

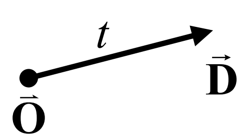
Ray Tracing Architecture



Ray-Plane Intersection

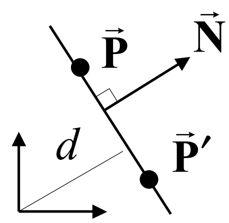
Ray: $\vec{\mathbf{P}} = \vec{\mathbf{O}} + t\vec{\mathbf{D}}$

$0 \le t < \infty$



Plane: $(\vec{\mathbf{P}} - \vec{\mathbf{P}}') \bullet \vec{\mathbf{N}} = 0$

$ax + by + cz + d = 0$



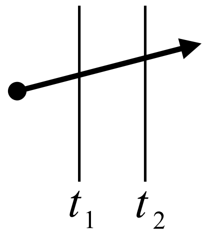
Solve for intersection $(\vec{\mathbf{P}} - \vec{\mathbf{P}}') \bullet \vec{\mathbf{N}} = (\vec{\mathbf{O}} + t\vec{\mathbf{D}} - \vec{\mathbf{P}}') \bullet \vec{\mathbf{N}} = 0$

Substitute ray eqn into plane equation

$$t = -\frac{(\vec{\mathbf{O}} - \vec{\mathbf{P}}') \bullet \vec{\mathbf{N}}}{\vec{\mathbf{D}} \bullet \vec{\mathbf{N}}}$$

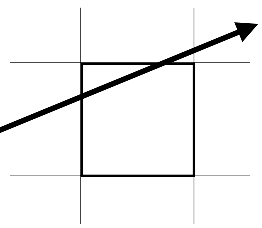
Ray-Polyhedra

Ray-Slab

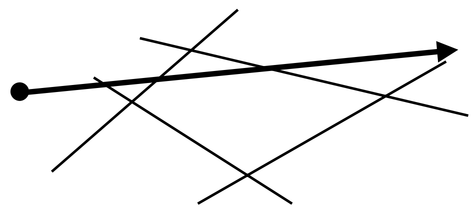


Note: Procedural Geometry!

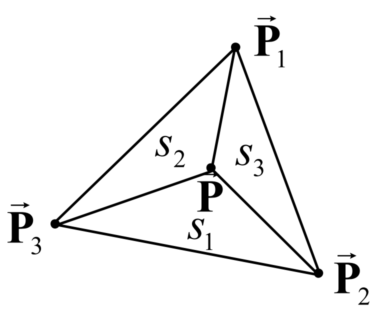
Ray-Box



Ray-Convex Polyhedra



Ray-Triangle Intersection



Barycentric coordinates

$$\vec{\mathbf{P}} = s_1\vec{\mathbf{P}}_1 + s_2\vec{\mathbf{P}}_2 + s_3\vec{\mathbf{P}}_3$$

Inside criteria

$$0 \le s_1 \le 1$$

$$0 \le s_2 \le 1$$

$$0 \le s_3 \le 1$$

$$s_1 + s_2 + s_3 \le 1$$

Ray-Triangle Intersection

$$\vec{\mathbf{P}} = s_1\vec{\mathbf{P}}_1 + s_2\vec{\mathbf{P}}_2 + s_3\vec{\mathbf{P}}_3 \Rightarrow \begin{bmatrix}\mathbf{P}_1 & \mathbf{P}_2 & \mathbf{P}_3\end{bmatrix}\begin{bmatrix}s_1\\ s_2\\ s_3\end{bmatrix} = \begin{bmatrix}\mathbf{P}\end{bmatrix}$$

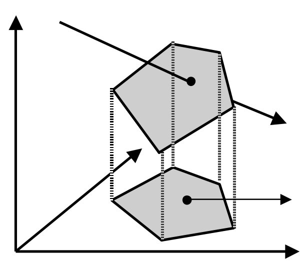
$$s_1 = \frac{\begin{vmatrix}\mathbf{P} & \mathbf{P}_2 & \mathbf{P}_3\end{vmatrix}}{\begin{vmatrix}\mathbf{P}_1 & \mathbf{P}_2 & \mathbf{P}_3\end{vmatrix}} = \mathbf{P} \bullet \frac{\mathbf{P}_2 \times \mathbf{P}_3}{\Delta}$$

$$s_2 = \frac{\begin{vmatrix}\mathbf{P}_1 & \mathbf{P} & \mathbf{P}_3\end{vmatrix}}{\begin{vmatrix}\mathbf{P}_1 & \mathbf{P}_2 & \mathbf{P}_3\end{vmatrix}} = \mathbf{P} \bullet \frac{\mathbf{P}_3 \times \mathbf{P}_1}{\Delta}$$

$$s_3 = \frac{\begin{vmatrix}\mathbf{P}_1 & \mathbf{P}_2 & \mathbf{P}\end{vmatrix}}{\begin{vmatrix}\mathbf{P}_1 & \mathbf{P}_2 & \mathbf{P}_3\end{vmatrix}} = \mathbf{P} \bullet \frac{\mathbf{P}_1 \times \mathbf{P}_2}{\Delta}$$

Ray-Polygon Intersection

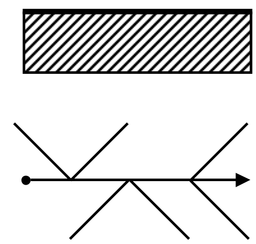
1. Find intersection with plane of support
2. Test whether point is inside 3d polygon
   a. Project onto xy plane (actually use max N coord.)
   b. Test whether point is inside 2d polygon



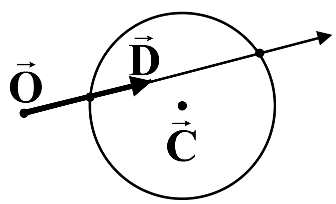
Point in Polygon

```
inside(vert v[], int n, float x, float y)
{
  int cross=0; float x0, y0, x1, y1;

  x0 = v[n-1].x - x;
  y0 = v[n-1].y - y;
  while( n-- ) {
    x1 = v->x - x;
    y1 = v->y - y;
    if( y0 > 0 ) {
      if( y1 <= 0 )
        if( x1*y0 > y1*x0 ) cross++;
    }
    else {
      if( y1 > 0 )
        if( x0*y1 > y0*x1 ) cross++;
    }
    x0 = x1; y0 = y1; v++;
  }
  return cross & 1;
}
```



Ray-Sphere Intersection

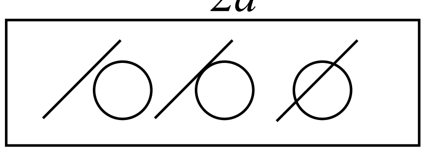


Ray: $\vec{\mathbf{P}} = \vec{\mathbf{O}} + t\vec{\mathbf{D}}$

Sphere: $(\vec{\mathbf{P}} - \vec{\mathbf{C}})^2 - R^2 = 0$

$$(\vec{\mathbf{O}} - t\vec{\mathbf{D}} - \vec{\mathbf{C}})^2 - R^2 = 0$$

$$t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$



$$at^2 + bt + c = 0$$
$$a = \vec{\mathbf{D}}^2 = 1$$
$$b = 2(\vec{\mathbf{O}} - \vec{\mathbf{C}}) \bullet \vec{\mathbf{D}}$$
$$c = (\vec{\mathbf{O}} - \vec{\mathbf{C}})^2 - R^2$$

Geometric Methods

Methods

- **Find normal**
- **Find surface parameters**

E.g. Sphere

Normal $\vec{\mathbf{N}} = \vec{\mathbf{P}} - \vec{\mathbf{O}}$

Parameters

$$x = \sin\theta\cos\phi \qquad \phi = \tan^{-1}(x, y)$$
$$y = \sin\theta\sin\phi \qquad \theta = \cos^{-1} z$$
$$z = \cos\theta$$

Ray-Implicit Surface Intersection

$$f(x, y, z) = 0$$

$$x = x_0 + x_1 t$$
$$y = y_0 + y_1 t$$
$$z = z_0 + z_1 t$$

$$f^*(t) = 0$$

1. Substitute ray equation

2. Find *positive*, *real* roots

Univariate root finding

- **Newton's method**
- ***Regula-falsi***
- **Interval methods**
- **Heuristics**

Ray-Algebraic Surface Intersection

$$p_n(x,y,z)=0$$

$$x = x_0 + x_1 t$$
$$y = y_0 + y_1 t$$
$$z = z_0 + z_1 t$$

$$p_n^*(t)=0$$

Degree *n*

- *Linear*: Plane
- *Quadric*: Spheres, Cylinders, Cones, Paraboloids, Hyperboloids
- *Quartic*: Tori

Polynomial root finding

- Quadratic, cubic, quartic
- Bezier/Bernoulli basis
- Descarte's rule of signs
- Sturm sequences

History

Polygons	Appel '68
Quadrics, CSG	Goldstein and Nagel '71
Tori	Roth '82
Bicubic patches	Whitted '80, Kajiya '82
Superquadrics	Edwards and Barr '83
Algebraic surfaces	Hanrahan '82
Swept surfaces	Kajiya '83, van Wijk '84
Fractals	Kajiya '83
Height fields	Coquillart & Gangnet '84, Musgrave '88
Deformations	Barr '86
Subdivision surfaces	Kobbelt, Daubert, Siedel, '98