

Reflection Models

Tuesday

- Reflection models
- The reflection equation and the BRDF
- Ideal reflection, refraction and diffuse

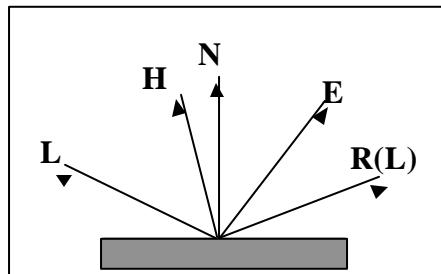
Today

- Glossy reflection models
- Rough surfaces
- Microfacets
- Self-shadowing
- Anisotropic reflection models

CS348B Lecture 10

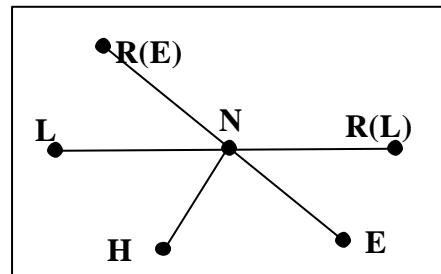
Pat Hanrahan, Spring 2000

Reflection Geometry



$$\cos q_i = \vec{\mathbf{L}} \cdot \vec{\mathbf{N}}$$

$$\cos q_r = \vec{\mathbf{R}} \cdot \vec{\mathbf{N}}$$



$$\cos q_g = \vec{\mathbf{E}} \cdot \vec{\mathbf{L}}$$

$$\cos q_s = \vec{\mathbf{E}} \cdot \mathbf{R}(\vec{\mathbf{L}}) = \mathbf{R}(\vec{\mathbf{E}}) \cdot \vec{\mathbf{L}}$$

$$\cos q_s' = \vec{\mathbf{H}} \cdot \vec{\mathbf{N}}$$

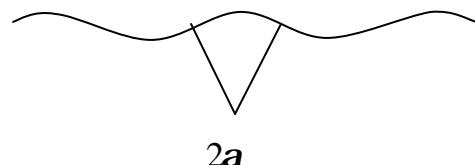
CS348B Lecture 10

Pat Hanrahan, Spring 2000

Reflection of the Sun from the Sea



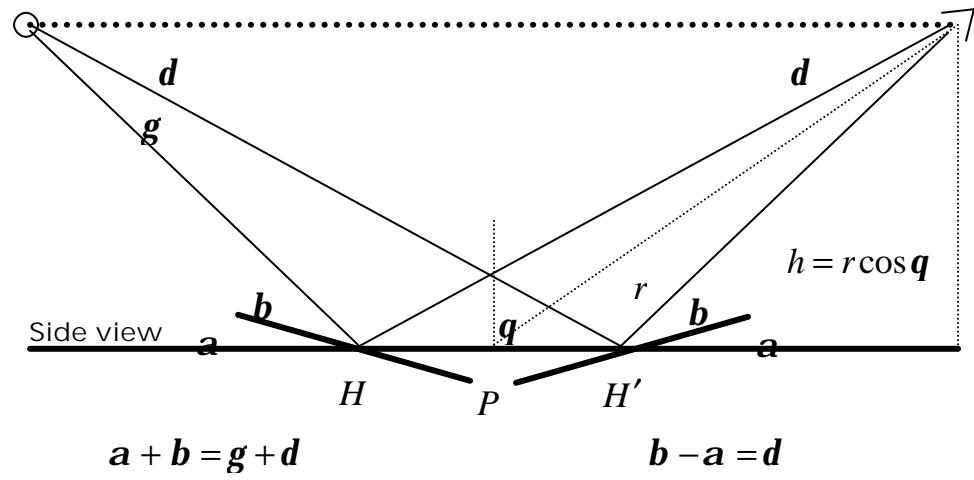
Minnaert p. 28



CS348B Lecture 10

Pat Hanrahan, Spring 2000

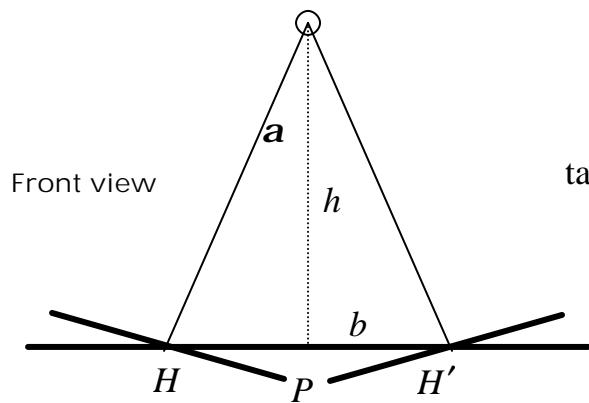
Reflection Angles



CS348B Lecture 10

Pat Hanrahan, Spring 2000

Reflection Angles



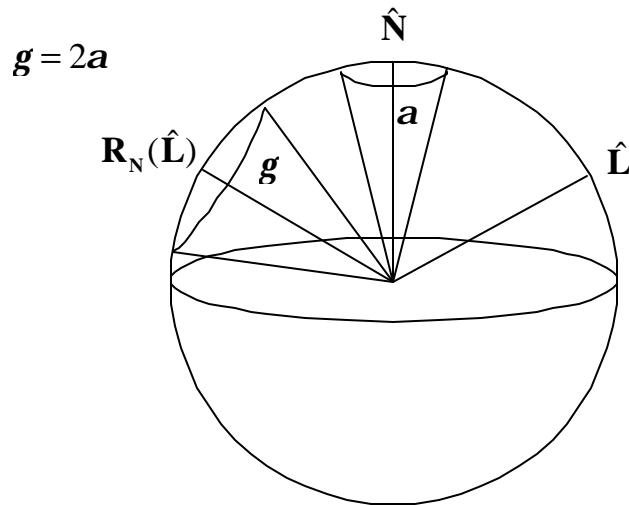
$$\tan y = \frac{b}{r} = \tan a \cos q$$

$$\tan a = \frac{b}{h}$$

CS348B Lecture 10

Pat Hanrahan, Spring 2000

Analysis on the Sphere

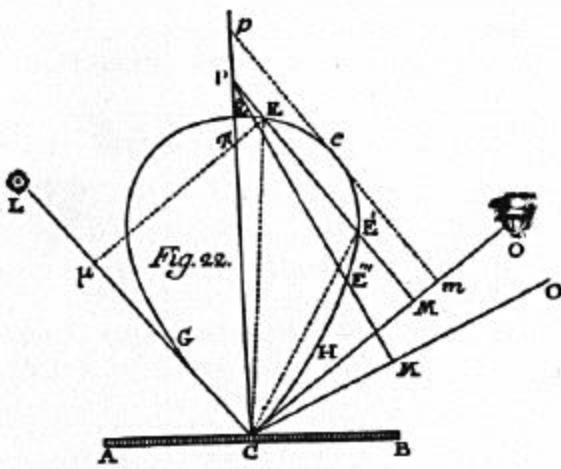


$$g = 2a$$

CS348B Lecture 10

Pat Hanrahan, Spring 2000

Bouguer's "little faces"

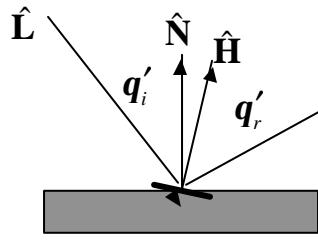


P. Bouguer, *Treatise on Optics*, 1760

CS348B Lecture 10

Pat Hanrahan, Spring 2000

Torrance-Sparrow Model



$$\begin{aligned} d^2\Phi_h &= L_i(\mathbf{w}_i) \cos q'_i d\mathbf{w}_i dA' \\ &= L_i(\mathbf{w}_i) \cos q'_i d\mathbf{w}_i D(\mathbf{w}_h) d\mathbf{w}_h dA \\ d^2\Phi_r &= d^2\Phi_m \end{aligned}$$

$$dA' = D(\mathbf{w}_h) d\mathbf{w}_h dA$$

$$d^2\Phi_r = dL_r(\mathbf{w}_i \rightarrow \mathbf{w}_r) \cos q_r d\mathbf{w}_r dA$$

$$\cos q_i = \hat{\mathbf{L}} \cdot \hat{\mathbf{N}}$$

$$dL_r(\mathbf{w}_i \rightarrow \mathbf{w}_r) \cos q_r d\mathbf{w}_r dA$$

$$\cos q'_i = \hat{\mathbf{L}} \cdot \hat{\mathbf{H}}$$

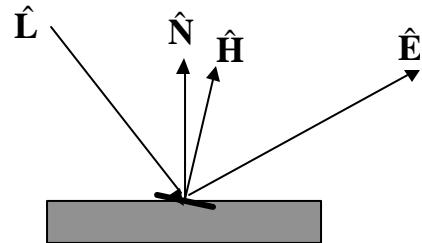
$$= L_i(\mathbf{w}_i) \cos q'_i d\mathbf{w}_i D(\mathbf{w}_h) d\mathbf{w}_h dA$$

$$d\mathbf{w}'_i = d\mathbf{w}_i$$

CS348B Lecture 10

Pat Hanrahan, Spring 2000

Torrance-Sparrow Model

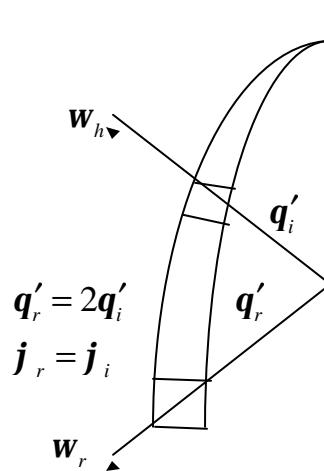
$$\begin{aligned}
 f_r(\mathbf{w}_i \rightarrow \mathbf{w}_r) &= \frac{dL_r(\mathbf{w}_i \rightarrow \mathbf{w}_r)}{dE_i(\mathbf{w}_i)} \\
 &= \frac{L_i(\mathbf{w}_i) \cos \mathbf{q}' d\mathbf{w}_i D(\mathbf{w}_h) d\mathbf{w}_h dA}{(\cos \mathbf{q}_r d\mathbf{w}_r dA)(L_i(\mathbf{w}_i) \cos \mathbf{q}_i d\mathbf{w}_i)} \\
 &= \frac{D(\mathbf{w}_h)}{\cos \mathbf{q}_i \cos \mathbf{q}_r} \cos \mathbf{q}' \frac{d\mathbf{w}_h}{d\mathbf{w}_r} \\
 &= \frac{D(\mathbf{w}_h)}{4 \cos \mathbf{q}_i \cos \mathbf{q}_r}
 \end{aligned}$$


The diagram illustrates the geometry of the Torrance-Sparrow model. A light source \hat{L} is positioned above a gray rectangular surface. At a point on the surface, a vertical arrow labeled \hat{N} represents the normal vector. A curved surface labeled \hat{H} represents the hemisphere above the point. A horizontal arrow labeled \hat{E} indicates the direction of the reflected ray.

CS348B Lecture 10

Pat Hanrahan, Spring 2000

Solid Angle Distributions



The diagram shows a unit sphere centered at the origin. A vertical vector \mathbf{w}_h points from the center. A horizontal vector $\mathbf{q}'_r = 2\mathbf{q}'_i$ is shown. A vector $\mathbf{j}_r = \mathbf{j}_i$ is also shown. A horizontal vector \mathbf{w}_r is shown at the bottom left. A small rectangle is drawn on the sphere's surface.

$$\begin{aligned}
 d\mathbf{w}_r &= \sin \mathbf{q}'_r d\mathbf{q}'_r d\mathbf{j}_r \\
 &= (\sin 2\mathbf{q}'_i) 2d\mathbf{q}'_i d\mathbf{j}_i \\
 &= (2 \sin \mathbf{q}'_i \cos \mathbf{q}'_i) 2d\mathbf{q}'_i d\mathbf{j}_i \\
 &= 4 \cos \mathbf{q}'_i d\mathbf{w}_i
 \end{aligned}$$

$$\frac{d\mathbf{w}_h}{d\mathbf{w}_r} = \frac{1}{4 \cos \mathbf{q}'_i}$$

CS348B Lecture 10

Pat Hanrahan, Spring 2000

Normalizing Microfacet Distributions

$$dA' = D(\mathbf{w}_h) d\mathbf{w}_h dA$$

$$\int_{H^2} \cos \mathbf{q}_h dA' = \int_{H^2} D(\mathbf{w}_h) d\mathbf{w}_h dA = dA$$

$$\int_{H^2} D(\mathbf{w}_h) \cos \mathbf{q}_h d\mathbf{w}_h = 1$$

CS348B Lecture 10

Pat Hanrahan, Spring 2000

Microfacet Distribution Functions

Isotropic distributions $D(\mathbf{w}) = D(\mathbf{a})$

Characterize by half-angle $D(\mathbf{b}) = \frac{1}{2}$

Examples:

■ Blinn $D_1(\mathbf{a}) = \cos^{c_1} \mathbf{a}$ $c_1 = \frac{\ln 2}{\ln \cos \mathbf{b}}$

■ Torrance-Sparrow $D_2(\mathbf{a}) = e^{-(c_2 \mathbf{a})^2}$ $c_2 = \frac{\sqrt{2}}{\mathbf{b}}$

■ Trowbridge-Reitz $D_3(\mathbf{a}) = \frac{c_3^2}{(1 - c_3^2) \cos^2 \mathbf{a} - 1}$ $c_3 = \left(\frac{\cos^2 \mathbf{b} - 1}{\cos^2 \mathbf{b} - \sqrt{2}} \right)^{\frac{1}{2}}$

CS348B Lecture 10

Pat Hanrahan, Spring 2000

Self-Shadowing: V-Groove Model

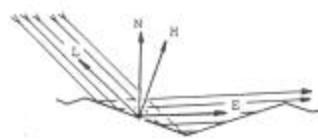
Assumptions (Torrance-Sparrow)

1. Symmetric, longitudinal, isotropically-distributed

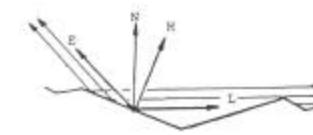
2. Upper edges lie in plane $G = \min(G_a, G_b, G_c)$



$$G_a = 1$$



$$G_b = \frac{2(\mathbf{N} \cdot \mathbf{H})(\mathbf{N} \cdot \mathbf{E})}{(\mathbf{H} \cdot \mathbf{E})}$$

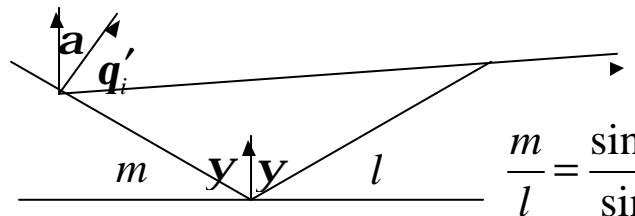


$$G_c = \frac{2(\mathbf{N} \cdot \mathbf{H})(\mathbf{N} \cdot \mathbf{L})}{(\mathbf{H} \cdot \mathbf{L})}$$

CS348B Lecture 10

Pat Hanrahan, Spring 2000

Self-Shadowing: V-Groove Model



$$\frac{m}{l} = \frac{\sin m}{\sin l}$$

$$\begin{aligned}\sin l &= \cos q'_i \\ \cos l &= \sin q'_i \\ \sin y &= \cos a \\ \cos y &= \sin a\end{aligned}$$

$$\begin{aligned}\sin m &= \sin l + 2y \\ &= \sin l \cos 2y + \cos l \sin 2y \\ &= \cos q'_i \cos 2y + \sin q'_i \sin 2y \\ &= \cos q'_i (1 - 2 \sin^2 y) + \sin q'_i 2 \cos y \sin y \\ &= \cos q'_i (1 - 2 \cos^2 a) + \sin q'_i 2 \cos a \sin a \\ &= \cos q'_i - 2 \cos a (\cos a \cos q'_i - \sin a \sin q'_i) \\ &= \cos q'_i - 2 \cos a \cos(a + q'_i) \\ &= \cos q'_i - 2 \cos a \cos q_r \\ &= \mathbf{H} \cdot \mathbf{E} - 2(\mathbf{N} \cdot \mathbf{H})(\mathbf{N} \cdot \mathbf{E})\end{aligned}$$

$$\begin{aligned}G &= 1 - \frac{m}{l} \\ &= 1 - \frac{\sin m}{\sin l} \\ &= \frac{\mathbf{H} \cdot \mathbf{E} - \mathbf{H} \cdot \mathbf{E} + 2(\mathbf{N} \cdot \mathbf{H})(\mathbf{N} \cdot \mathbf{E})}{\mathbf{H} \cdot \mathbf{E}} \\ &= \frac{2(\mathbf{N} \cdot \mathbf{H})(\mathbf{N} \cdot \mathbf{E})}{\mathbf{H} \cdot \mathbf{E}}\end{aligned}$$

CS348B Lecture 10

Pat Hanrahan, Spring 2000

Gaussian Rough Surface

Beckmann

$$p(z) = \frac{1}{\sqrt{2\pi s^2}} e^{-\frac{z^2}{2s^2}}$$

$$D(\alpha) = \frac{1}{\sqrt{\pi m^2 \cos^2 \alpha}} e^{-\frac{\tan^2 \alpha}{m^2}}$$

Smith

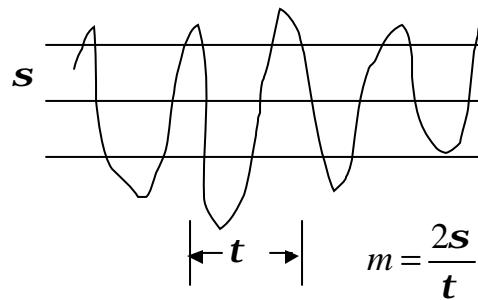
Derives shadowing function probabilistically

Self-consistency condition

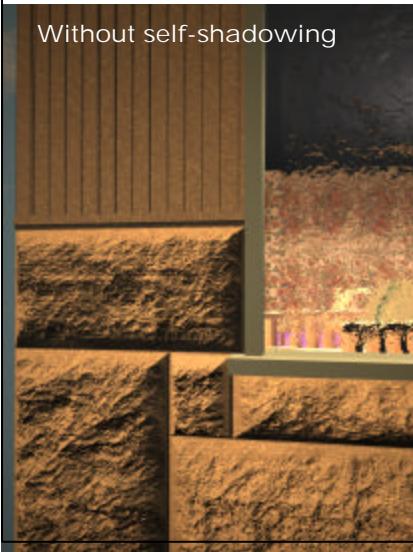
The sum of the areas of the illuminated surface projected onto the plane normal to the direction of incidence is independent of the roughness of the surface, and equal to the projected area of the underlying mean plane.

CS348B Lecture 10

Pat Hanrahan, Spring 2000



Shadows on Rough Surfaces



Without self-shadowing



With self-shadowing

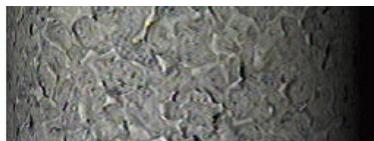
CS348B Lecture 10

Pat Hanrahan, Spring 2000

S. Nayar's BTF Experiments

Complex interplay between texture and brdf

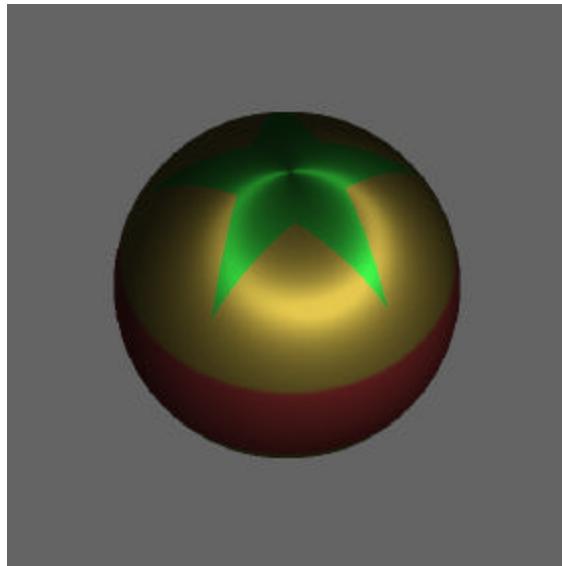
Self-shadowing a major effect



CS348B Lecture 10

Pat Hanrahan, Spring 2000

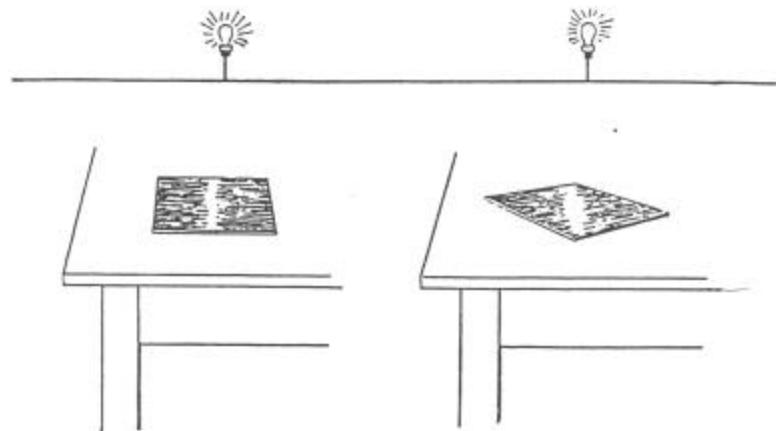
Anisotropic Reflection



CS348B Lecture 10

Pat Hanrahan, Spring 2000

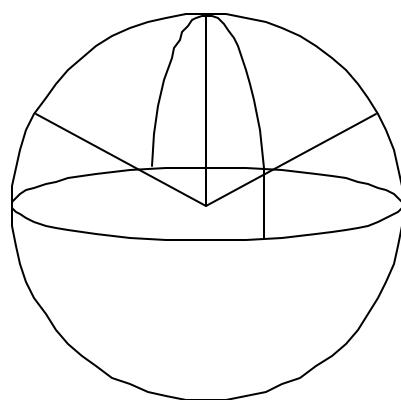
Anisotropic Reflection



CS348B Lecture 10

Pat Hanrahan, Spring 2000

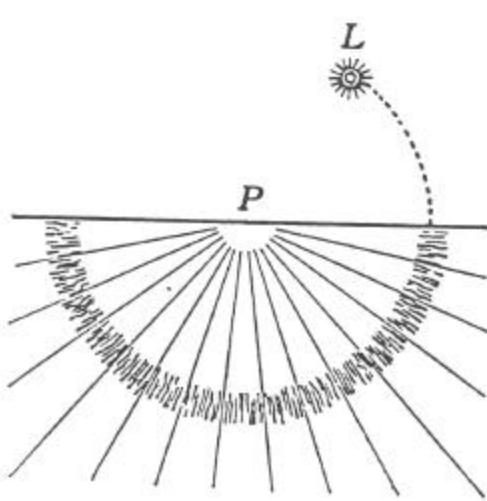
Anisotropic Reflection



CS348B Lecture 10

Pat Hanrahan, Spring 2000

Anisotropic Reflection



CS348B Lecture 10

Pat Hanrahan, Spring 2000