Optics I: lenses and apertures

CS 178, Spring 2013

Begun 4/9/13, finished 4/11/13.

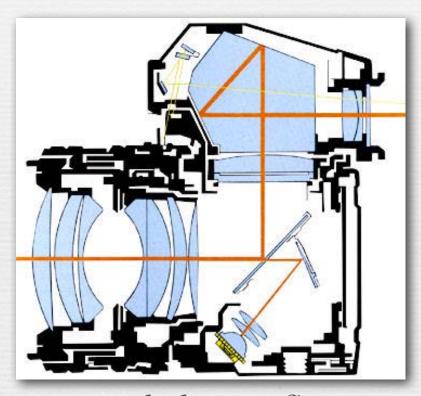


Marc Levoy
Computer Science Department
Stanford University

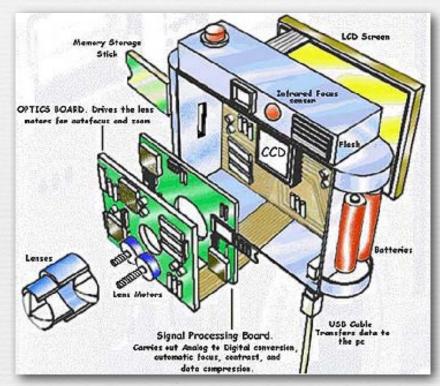
Outline

- why study lenses?
- ♦ thin lenses
 - graphical constructions, algebraic formulae
- thick lenses
 - center of perspective, lens as $3D \rightarrow 3D$ transformation
- depth of field
- * aberrations & distortion
- vignetting, glare, and other lens artifacts
- diffraction and lens quality
- ◆ special lenses
 - telephoto, zoom

Cameras and their lenses

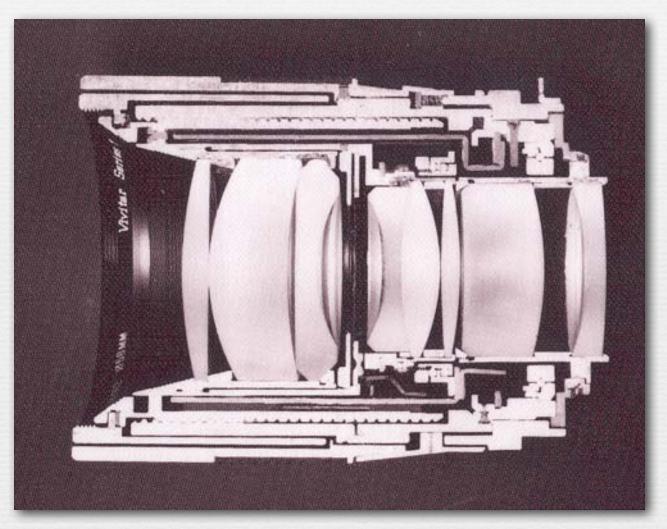


single lens reflex (SLR) camera



digital still camera (DSC), i.e. point-and-shoot

Cutaway view of a real lens



Vivitar Series 1 90mm f/2.5 Cover photo, Kingslake, *Optics in Photography*

Lens quality varies

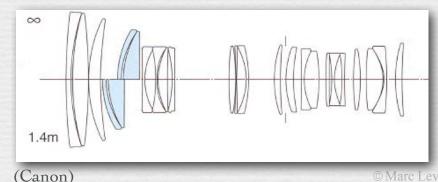
- ♦ Why is this toy so expensive?
 - EF 70-200mm f/2.8L IS USM
 - \$1700



- ♦ Why is it better than this toy?
 - EF 70-300mm f/4-5.6 IS USM
 - \$550



◆ And why is it so complicated?





Stanford Big Dish Panasonic GF1 Panasonic 45-200/4-5.6 zoom, at 200mm f/4.6 \$300

Leica 90mm/2.8 Elmarit-M prime, at f/4 \$2000

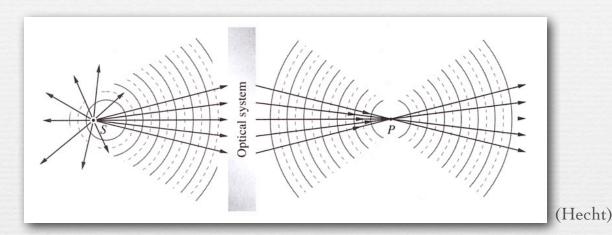
Zoom lens versus prime lens



Canon 100-400mm/4.5-5.6 zoom, at 300mm and f/5.6 \$1600

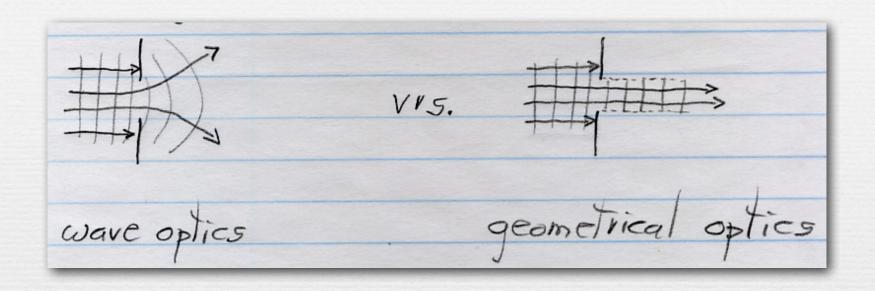
Canon 300mm/2.8 prime, at f/5.6 \$4300

Physical versus geometrical optics



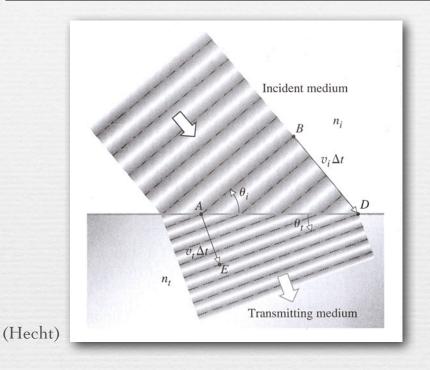
- light can be modeled as traveling waves
- ◆ the perpendiculars to these waves can be drawn as rays
- ♦ diffraction causes these rays to bend, e.g. at a slit
- → geometrical optics assumes
 - $\lambda \rightarrow 0$
 - no diffraction
 - in free space, rays are straight (a.k.a. rectilinear propagation)

Physical versus geometrical optics (contents of whiteboard)

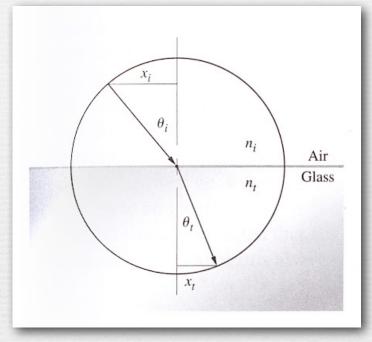


- in geometrical optics, we assume that rays do not bend as they pass through a narrow slit
- \star this assumption is valid if the slit is much larger than the wavelength, represented on the previous slide by the limit $\lambda \to 0$
- physical optics is a.k.a. wave optics

Snell's law of refraction



- as waves change
 speed at an interface,
 they also change direction
- \bullet index of refraction n_t is defined as



$$\frac{x_i}{x_t} = \frac{\sin \theta_i}{\sin \theta_t} = \frac{n_t}{n_i}$$

 $\frac{\text{speed of light in a vacuum}}{\text{speed of light in medium } t}$

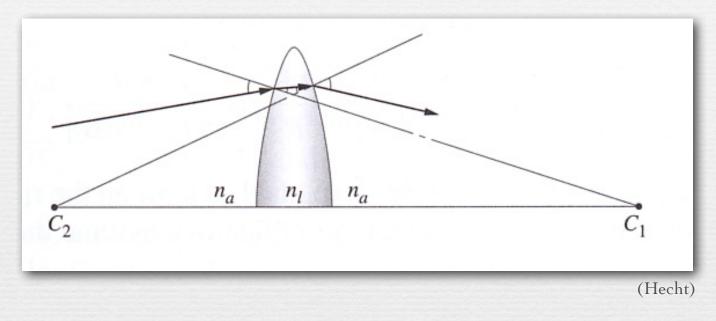
Typical refractive indices (n)

- + air = ~1.0
- → water = 1.33
- → glass = 1.5 1.8



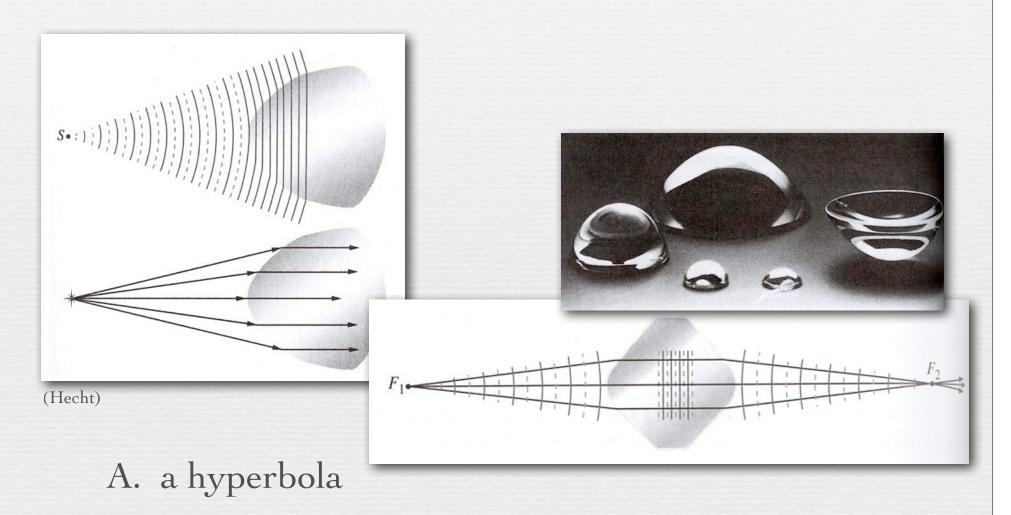
mirage due to changes in the index of refraction of air with temperature

Refraction in glass lenses



- when transiting from air to glass,
 light bends towards the normal
- when transiting from glass to air, light bends away from the normal
- ◆ light striking a surface perpendicularly does not bend

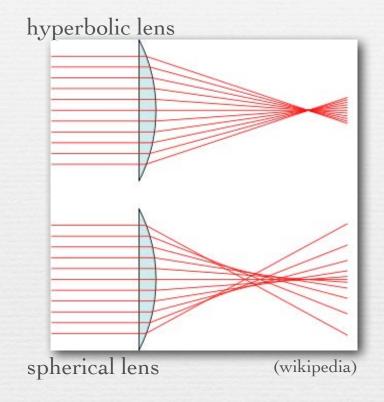
Q. What shape should a refractive interface be to make parallel rays converge to a point?



→ so lenses should be hyperbolic!

Spherical lenses





- two roughly fitting curved surfaces ground together will eventually become spherical
- spheres don't bring parallel rays to a point
 - this is called *spherical aberration*
 - nearly axial rays (paraxial rays) behave best

Examples of spherical aberration





Canon 135mm soft focus lens

(gtmerideth)

(Canon)

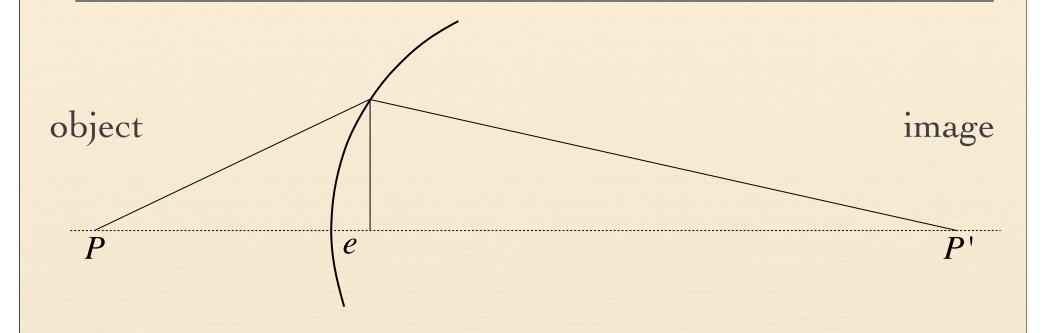








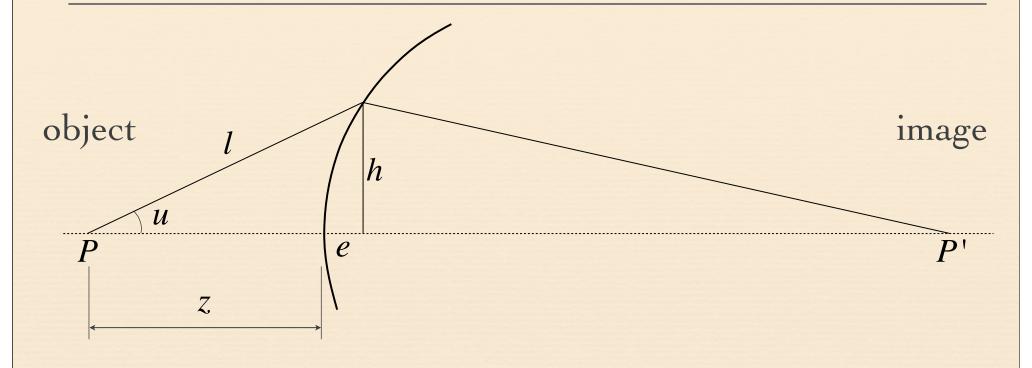
Paraxial approximation



 \star assume $e \approx 0$

Not responsible on exams for orange-tinted slides

Paraxial approximation



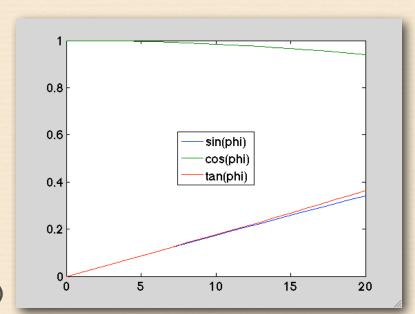
- \bullet assume $e \approx 0$
- → assume $sin u = h/l \approx u$ (for u in radians)
- → assume $\cos u \approx z/l \approx 1$
- ♦ assume $tan u \approx sin u \approx u$

The paraxial approximation is a.k.a. first-order optics

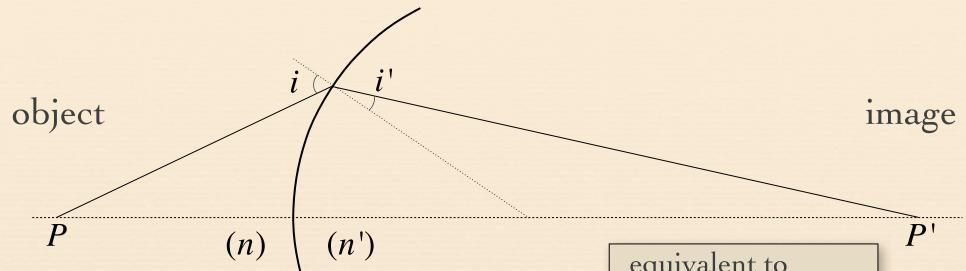
- * assume first term of $\sin \phi = \phi \frac{\phi^3}{3!} + \frac{\phi^5}{5!} \frac{\phi^7}{7!} + \dots$ • i.e. $\sin \phi \approx \phi$
- * assume first term of $\cos \phi = 1 \frac{\phi^2}{2!} + \frac{\phi^4}{4!} \frac{\phi^6}{6!} + \dots$
 - i.e. $\cos \phi \approx 1$
 - so $tan \phi \approx sin \phi \approx \phi$

these are the Taylor series for $\sin \phi$ and $\cos \phi$

(phi in degrees)



Paraxial focusing



Snell's law:

$$n \sin i = n' \sin i'$$

paraxial approximation:

$$n i \approx n' i'$$

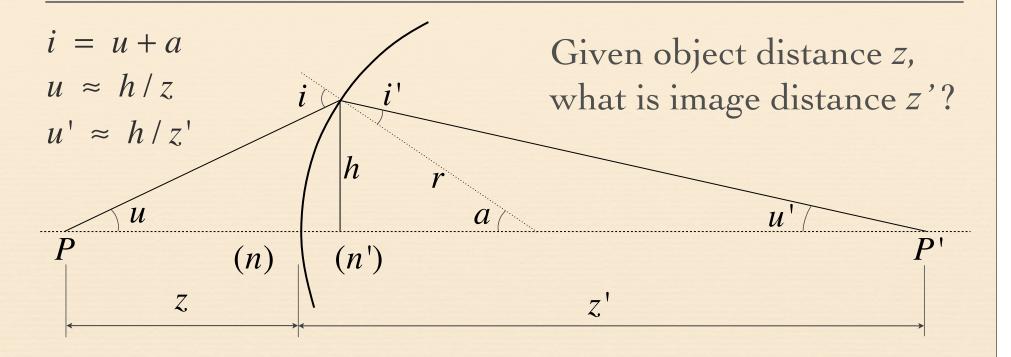
equivalent to

$$\frac{\sin \theta_i}{\sin \theta_t} = \frac{n_t}{n_i}$$

with

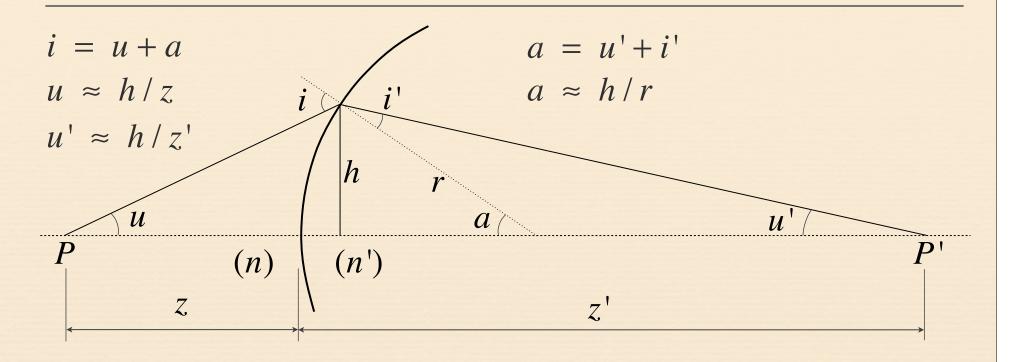
$$n = n_i$$
 for air
 $n' = n_t$ for glass
 i, i' in radians
 θ_i , θ_t in degrees

Paraxial focusing



$$ni \approx n'i'$$

Paraxial focusing



$$n(u+a) \approx n'(a-u')$$

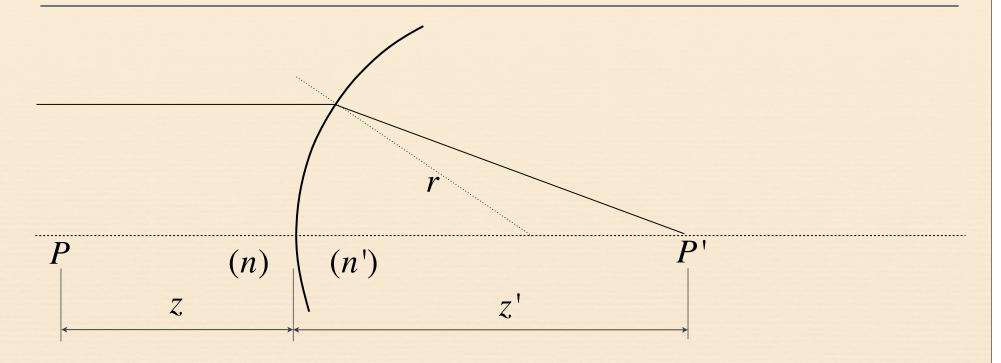
$$n(h/z+h/r) \approx n'(h/r-h/z')$$

$$n/z+n/r \approx n'/r-n'/z'$$

 $ni \approx n'i'$

♦ h has canceled out, so any ray from P will focus to P'

Focal length



What happens if z is ∞ ?

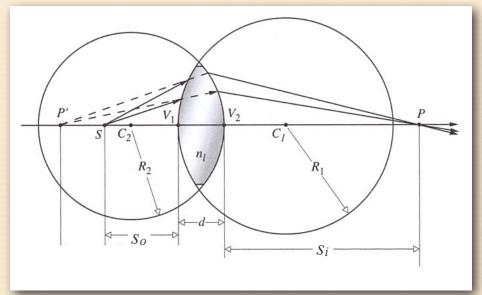
$$n/z + n/r \approx n'/r - n'/z'$$

$$n/r \approx n'/r - n'/z'$$

$$z' \approx (r n')/(n'-n)$$

Lensmaker's formula

 using similar derivations, one can extend these results to two spherical interfaces forming a lens in air



(Hecht, edited)

 \star as $d \to 0$ (thin lens approximation), we obtain the lensmaker's formula

$$\frac{1}{s_o} + \frac{1}{s_i} = (n_l - 1) \left(\frac{1}{R_1} - \frac{1}{R_2} \right)$$

Gaussian lens formula

◆ Starting from the lensmaker's formula

$$\frac{1}{s_o} + \frac{1}{s_i} = (n_l - 1) \left(\frac{1}{R_1} - \frac{1}{R_2} \right),$$
 (Hecht, eqn 5.15)

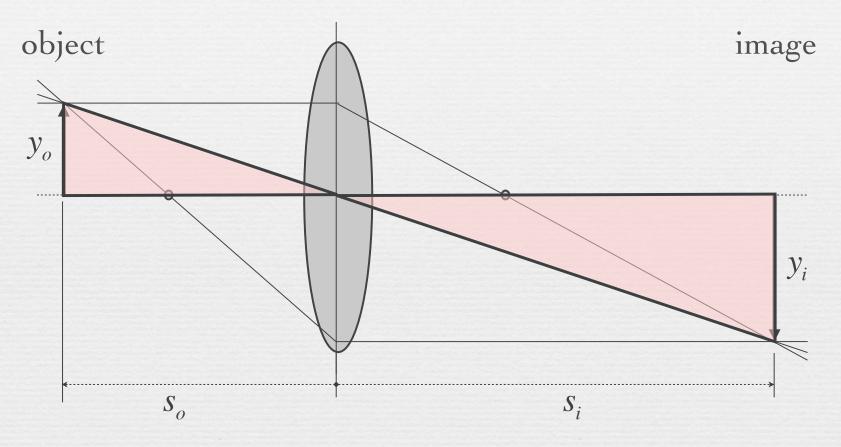
 \bullet and recalling that as object distance s_0 is moved to infinity, image distance s_i becomes focal length f_i , we get

$$\frac{1}{f_i} = (n_l - 1) \left(\frac{1}{R_1} - \frac{1}{R_2} \right).$$
 (Hecht, eqn 5.16)

◆ Equating these two, we get the Gaussian lens formula

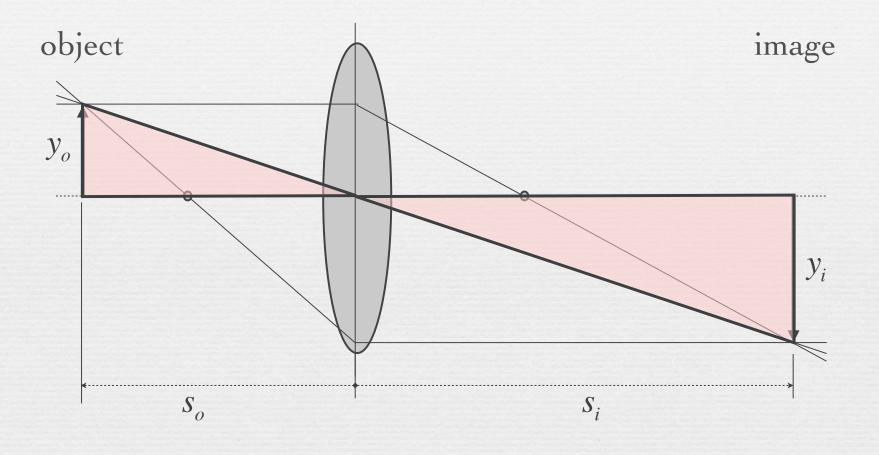
$$\frac{1}{s_o} + \frac{1}{s_i} = \frac{1}{f_i}$$
. (Hecht, eqn 5.17)

From Gauss's ray construction to the Gaussian lens formula



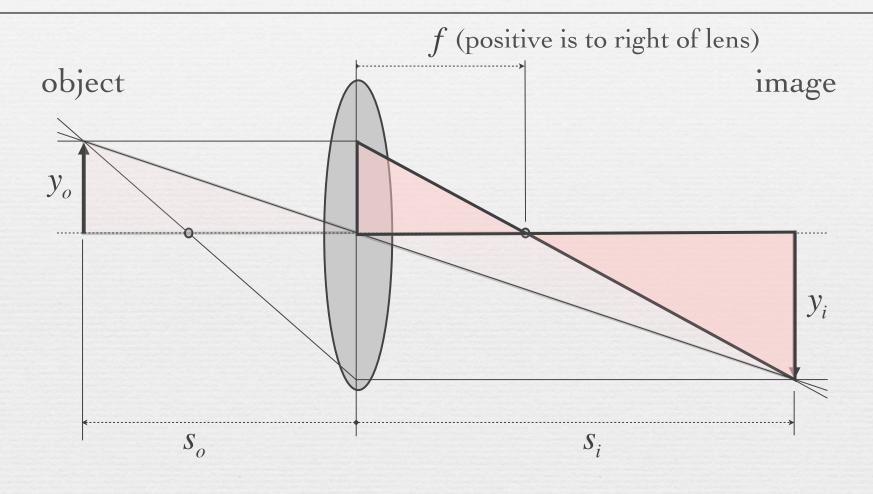
- \bullet positive s_i is rightward, positive s_o is leftward
- → positive y is upward

From Gauss's ray construction to the Gaussian lens formula



$$\frac{\left|y_{i}\right|}{y_{o}} = \frac{s_{i}}{s_{o}}$$

From Gauss's ray construction to the Gaussian lens formula

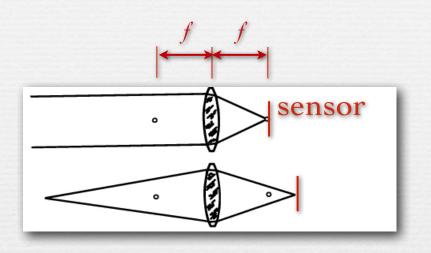


$$\frac{|y_i|}{y_o} = \frac{s_i}{s_o}$$
 and $\frac{|y_i|}{y_o} = \frac{s_i - f}{f}$

$$\frac{1}{s_o} + \frac{1}{s_i} = \frac{1}{f}$$

Changing the focus distance

 to focus on objects at different distances, move sensor relative to lens



(FLASH DEMO)

http://graphics.stanford.edu/courses/ cs178/applets/gaussian.html

$$\frac{1}{s_o} + \frac{1}{s_i} = \frac{1}{f}$$

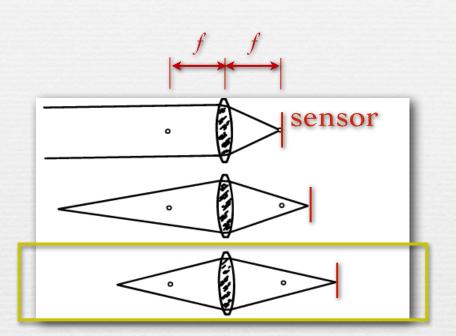
Changing the focus distance

to focus on objects
 at different distances,
 move sensor relative to lens

• at $s_o = s_i = 2f$ we have 1:1 imaging, because

$$\frac{1}{2f} + \frac{1}{2f} = \frac{1}{f}$$

In 1:1 imaging, if the sensor is 36mm wide, an object 36mm wide will fill the frame.



$$\frac{1}{s_o} + \frac{1}{s_i} = \frac{1}{f}$$

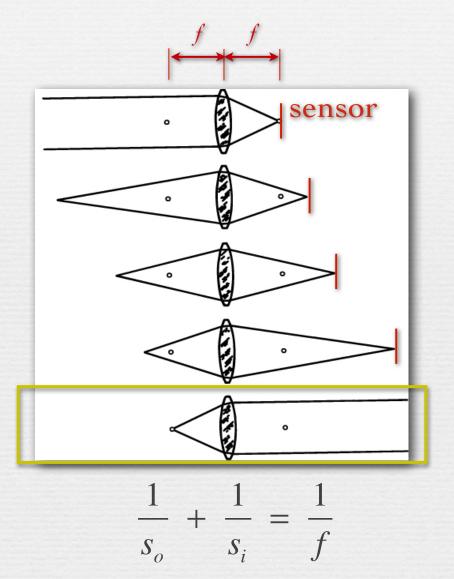
Changing the focus distance

to focus on objects
 at different distances,
 move sensor relative to lens

• at $s_o = s_i = 2f$ we have 1:1 imaging, because

$$\frac{1}{2f} + \frac{1}{2f} = \frac{1}{f}$$

 → can't focus on objects closer to lens than its focal length f

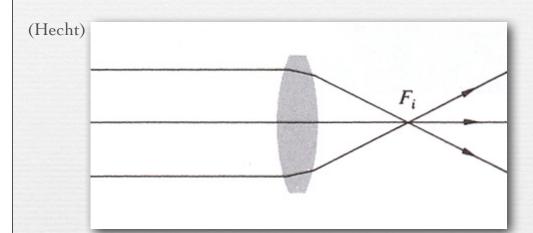


Recap

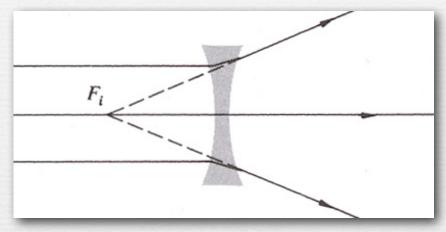
- approximations we sometimes make when analyzing lenses
 - geometrical optics instead of physical optics
 - spherical lenses instead of hyperbolic lenses
 - thin lens representation of thick optical systems
 - paraxial approximation of ray angles
- \star the Gaussian lens formula relates focal length f, object distance s_o , and image distance s_i
 - these settings, and sensor size, determine field of view
 - 1:1 imaging means $S_0 = S_i$ and both are $2 \times$ focal length
 - $S_0 = f$ is the minimum possible object distance for a lens

As a student pointed out by his question in class, one must be careful applying this last rule in practice, because s_0 may not be measured to the front of the barrel of a complex lens assembly, but to a plane you can't see inside the assembly. Also, many lenses can't crank out to the s_i that would enable the theoretically minimum s_0 , because it is mechanically difficult or might produce poorly corrected (aberrant) images.

Convex versus concave lenses



rays from a convex lens converge



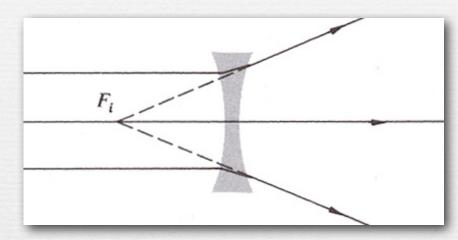
rays from a concave lens diverge

- → positive focal length f means parallel rays from the left converge to a point on the right
- → negative focal length f means parallel rays from the left converge to a point on the left (dashed lines above)

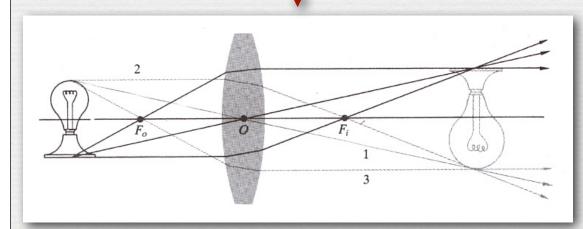
Convex versus concave lenses

(Hecht)

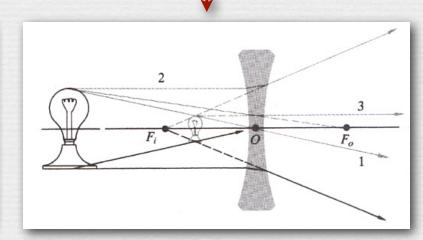
rays from a convex lens converge



rays from a concave lens diverge

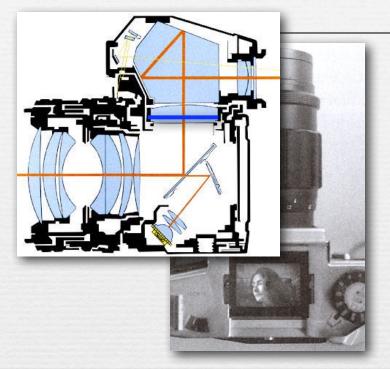


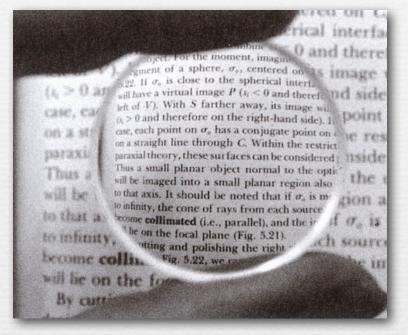
...producing a real image

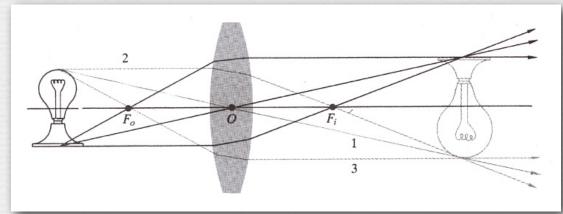


...producing a virtual image

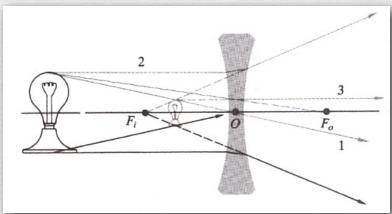
Convex versus concave lenses











...producing a virtual image

The power of a lens

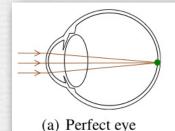
$$P = \frac{1}{f}$$

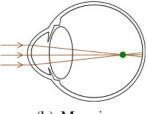
units are meters-1

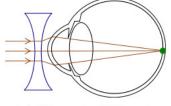
a.k.a. diopters

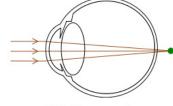
- my eyeglasses have the prescription
 - right eye: -0.75 diopters
 - left eye: -1.00 diopters
- Q. What's wrong with me?
- A. Myopia (nearsightedness)

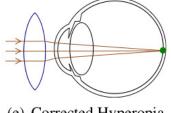
(Pamplona)











(b) Myopia

(c) Corrected Myopia

(d) Hyperopia

(e) Corrected Hyperopia

Combining two lenses

using focal lengths

$$\frac{1}{f_{tot}} = \frac{1}{f_1} + \frac{1}{f_2}$$

using diopters

$$P_{tot} = P_1 + P_2$$

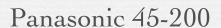
◆ example

$$\frac{1}{200mm} + \frac{1}{500mm} = \frac{1}{143mm}$$
 -or- $5.0 + 2.0 = 7.0$ diopters





- screw on to end of lens
- power is designated in diopters (usually)







→ changes longest focal length from 200mm to 143mm

$$\frac{1}{200mm} + \frac{1}{500mm} = \frac{1}{143mm}$$

- ◆ for a fixed image distance, it reduces the object distance
 - at f=200mm, this len's minimum object distance $s_o=1000$ mm
 - at these settings, its effective image distance must be

$$s_i = \frac{1}{\frac{1}{f} - \frac{1}{s_o}} = \frac{1}{\frac{1}{200mm} - \frac{1}{1000mm}} = 250mm$$

• with the closeup filter and the same settings of focal length and image distance, the in-focus object distance becomes

$$s_o = \frac{1}{\frac{1}{f} - \frac{1}{s_i}} = \frac{1}{\frac{1}{143mm} - \frac{1}{250mm}} = 334mm$$

 $3\times$

closer!

Canon

CZ6-5602

Close-up Lens / Bonnette Macro / Nahlinse / Lente Addizionale Diametro / lente de acercamiento / Оптический конвертер для макросъёмки / 近摄鏡片 / 近攝鏡 / CLOSE-UP 렌즈 / クローズアップレンズ

58mm Close-up Lens 500D

CANON INC.

Made in Japan/Fabriqué au Japon/Hecho en Japón/ Сделано в Японии/日本制造/日本製造

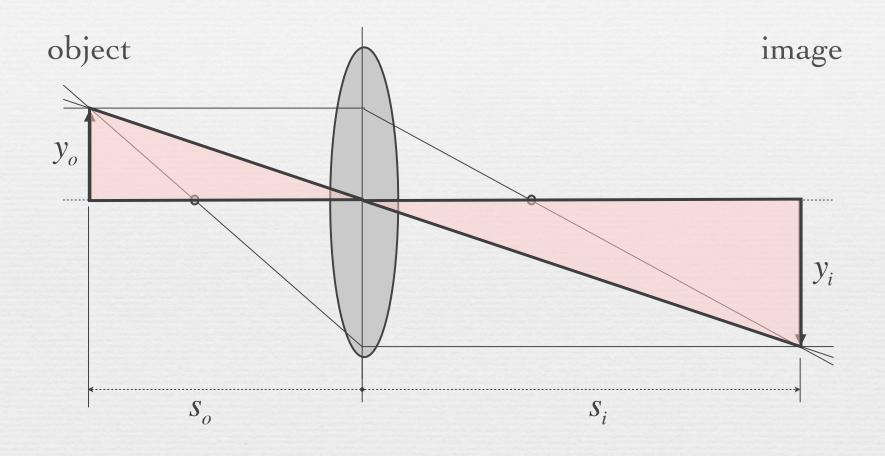
200mm lens & no closeup filter $s_0 = 1000$ mm



200mm lens & 500D closeup filter $s_0 = 334$ mm

poor man's macro lens

Magnification



$$M_T \triangleq \frac{y_i}{y_o} = -\frac{s_i}{s_o}$$

Canon

CZ6-5602

Close-up Lens / Bonnette Macro / Nahlinse / Lente Addizionale Diametro / lente de acercamiento / Оптический конвертер для макросъёмки / 近摄镜片 / 近攝鏡 / CLOSE-UP 렌즈 / クローズアップレンズ

58mm Close-up Lens 500D

CANON INC.

Made in Japan / Fabriqué au Japon / Hecho en Japón / Сделано в Японии / 日本制造 / 日本製造

> 200mm lens & no closeup filter $S_0 = 1000 \text{mm}$

$$M_T = -\frac{s_i}{s_o} = \frac{250}{1000} = -1:4$$

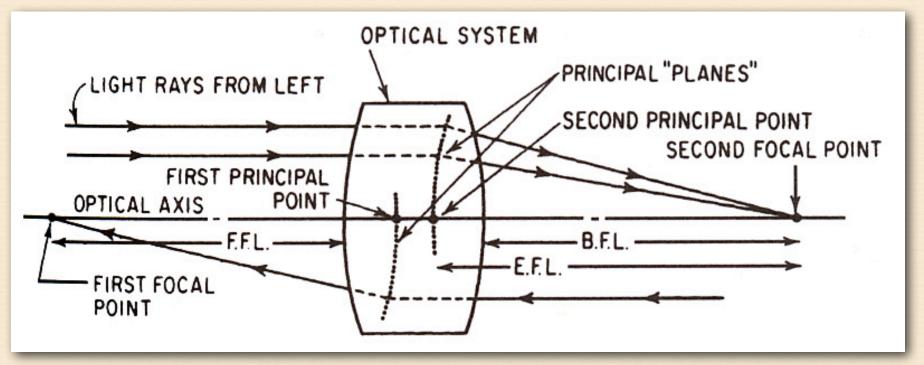


200mm lens & 500D closeup filter $S_0 = 334 \text{mm}$

$$M_T = -\frac{s_i}{s_o} = \frac{250}{1000} = -1:4$$
 $M_T = -\frac{s_i}{s_o} = \frac{250}{334} = -3:4$

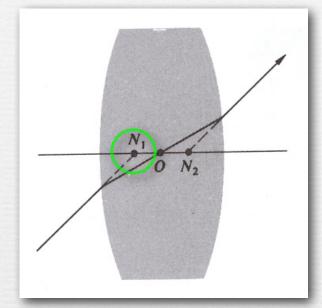
Thick lenses

an optical system may contain many lenses,
 but can be characterized by a few numbers



(Smith)

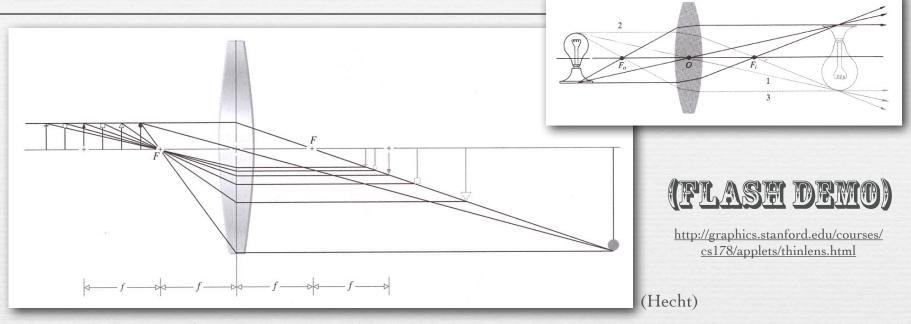
Center of perspective



(Hecht)

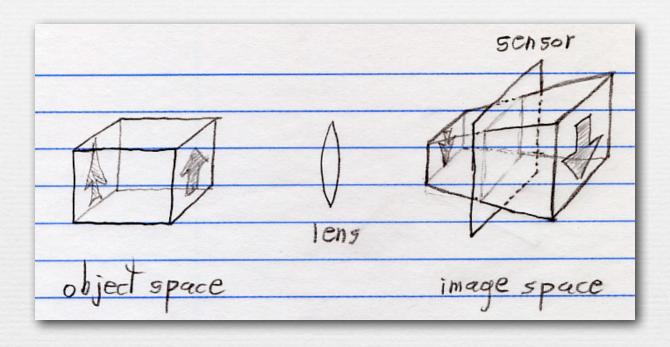
- in a thin lens, the *chief ray* from a point traverses the lens (through its optical center) without changing direction
- in a thick lens, the intersections of this ray with the optical axis are called the *nodal points*
- for a lens in air, these coincide with the principal points
- the first nodal point is the center of perspective

Lenses perform a 3D perspective transform



- ♦ lenses transform a 3D object to a 3D image; the sensor extracts a 2D slice from that image
- ◆ as an object moves linearly (in Z),
 its image moves non-proportionately (in Z)
- ◆ as you move a lens linearly relative to the sensor, the in-focus object plane moves non-proportionately
- → as you refocus a camera, the image changes size!

Lenses perform a 3D perspective transform (contents of whiteboard)



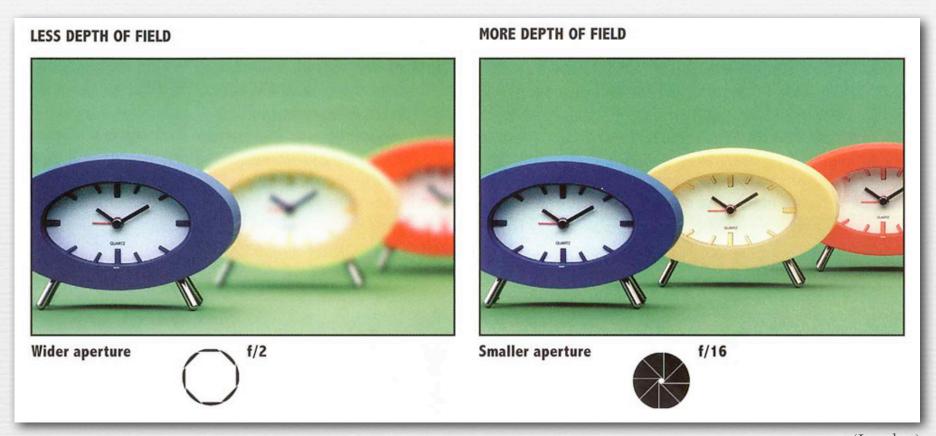
- → a cube in object space is transformed by a lens into a 3D frustum in image space, with the orientations shown by the arrows
- in computer graphics this transformation is modeled as a 4 × 4 matrix multiplication of 3D points expressed in 4D homogenous coordinates
- in photography a sensor extracts a 2D slice from the 3D frustum; on this slice some objects will be sharply focused; others may be blurry

Recap

- → more implications of the Gaussian lens formula
 - convex lenses make real images; concave make virtual images
 - the power of a lens (in diopters) is 1 over its focal length
 - when combining two lenses, add their powers
 - adding a closeup filter allows a smaller object distance
 - changing object and image distances changes magnification
- → lenses perform a 3D perspective transform of object space
 - an object's apparent size is inversely proportional to its distance
 - linear lens motions move the in-focus plane non-linearly
 - focusing a lens changes the image size (slightly)

Questions?

Depth of field

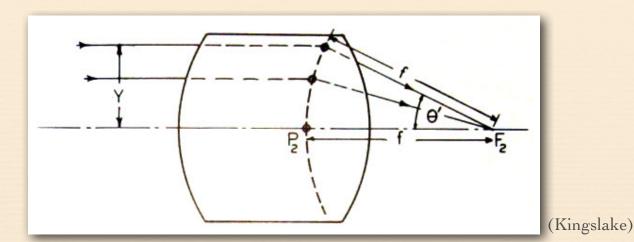


f (London)

$$N = \frac{J}{A}$$

→ lower N means a wider aperture and less depth of field

How low can N be?



◆ principal planes are the paraxial approximation of a

spherical "equivalent refracting surface"

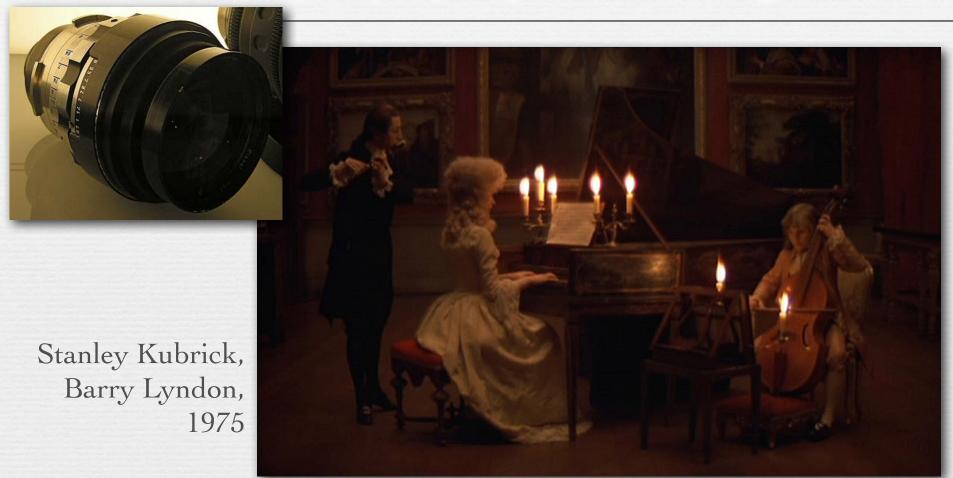
$$N = \frac{1}{2\sin\theta'}$$

- → lowest possible N in air is f/0.5
- → lowest N I've seen in an SLR is f/1.0



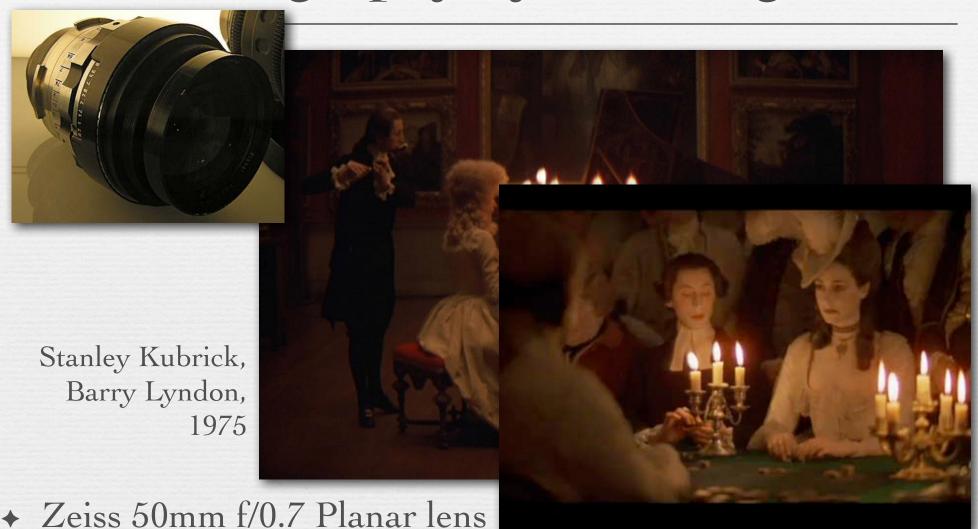
Canon EOS 50mm f/1.0 (discontinued)

Cinematography by candlelight



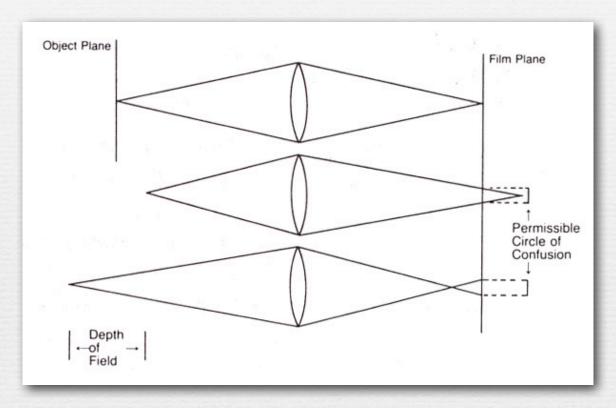
- → Zeiss 50mm f/0.7 Planar lens
 - originally developed for NASA's Apollo missions
 - very shallow depth of field in closeups (small object distance)

Cinematography by candlelight



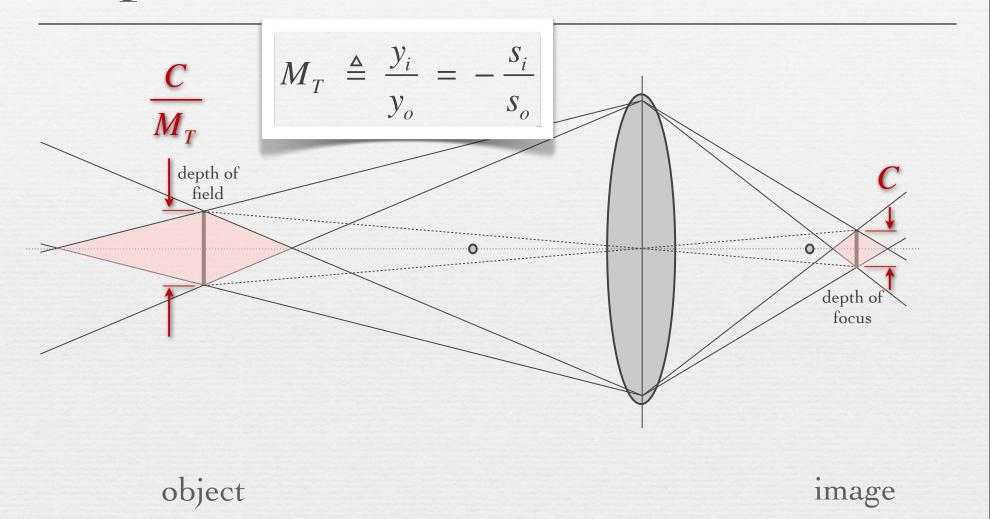
- - originally developed for NASA's Apollo missions
 - very shallow depth of field in closeups (small object distance)

Circle of confusion (C)

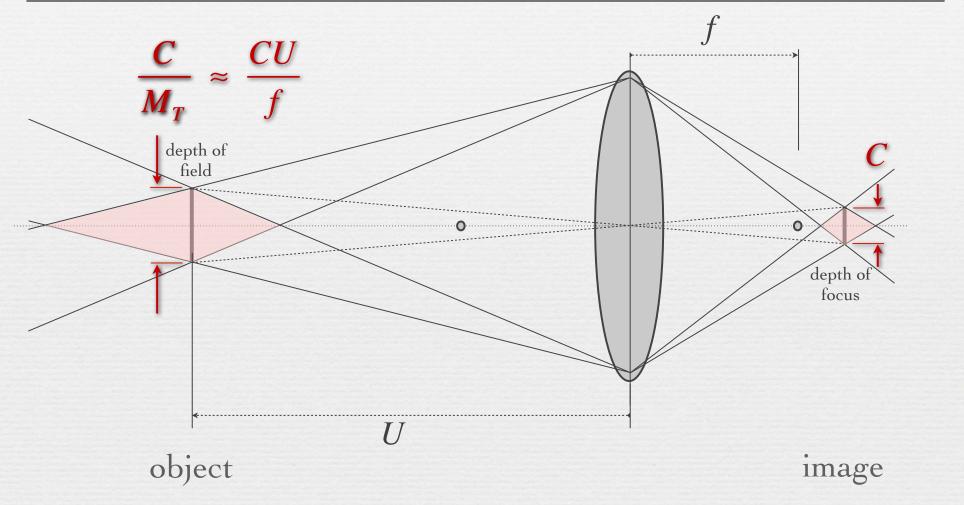


to derive C for a given situation, start from the smallest visual angle we can detect; we'll cover this later in the course

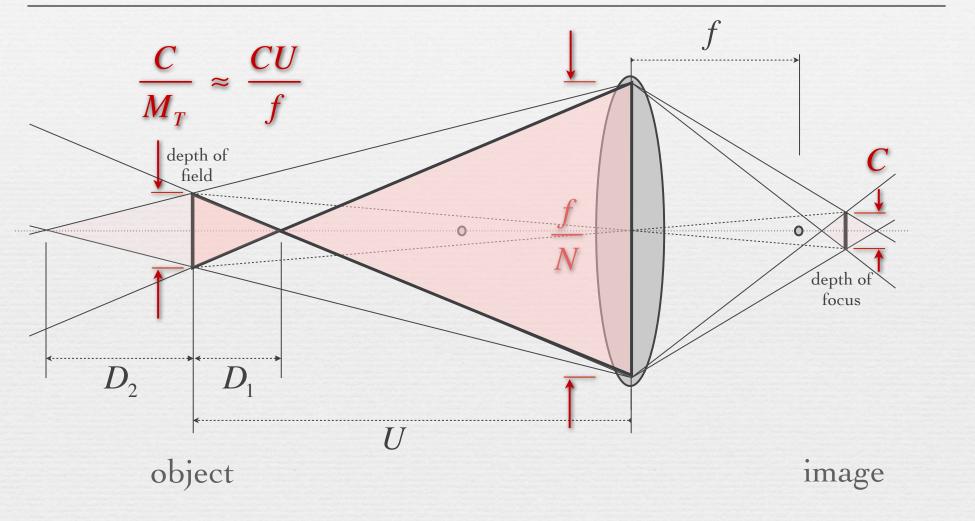
- ◆ C depends on sensing medium, reproduction medium, viewing distance, human vision,...
 - for print from 35mm film, 0.02mm (on negative) is typical
 - for high-end SLR, 6μ is typical (1 pixel)
 - larger if downsizing for web, or lens is poor



- → DoF is asymmetrical around the in-focus object plane
- → conjugate in object space is typically bigger than C



- ◆ DoF is asymmetrical around the in-focus object plane
- * conjugate in object space is typically bigger than C



$$\frac{D_1}{CU/f} = \frac{U - D_1}{f/N} \dots D_1 = \frac{NCU^2}{f^2 + NCU}$$

$$D_2 = \frac{NCU^2}{f^2 - NCU}$$

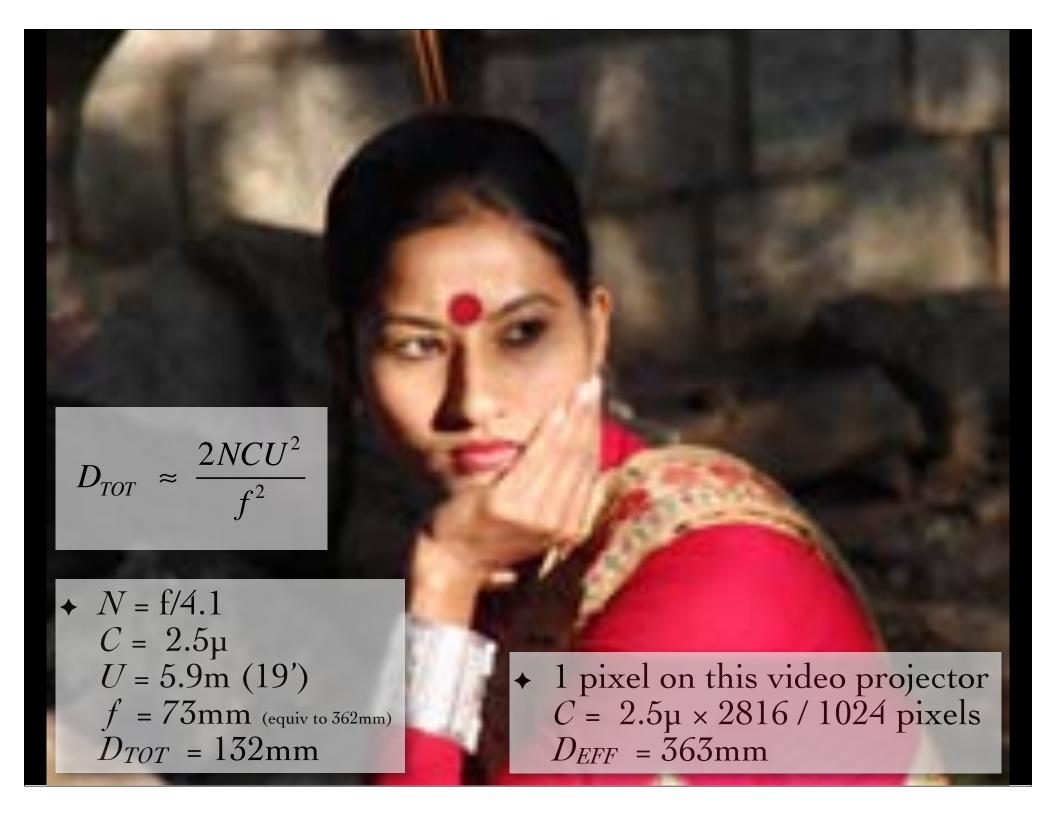
Marc Levoy

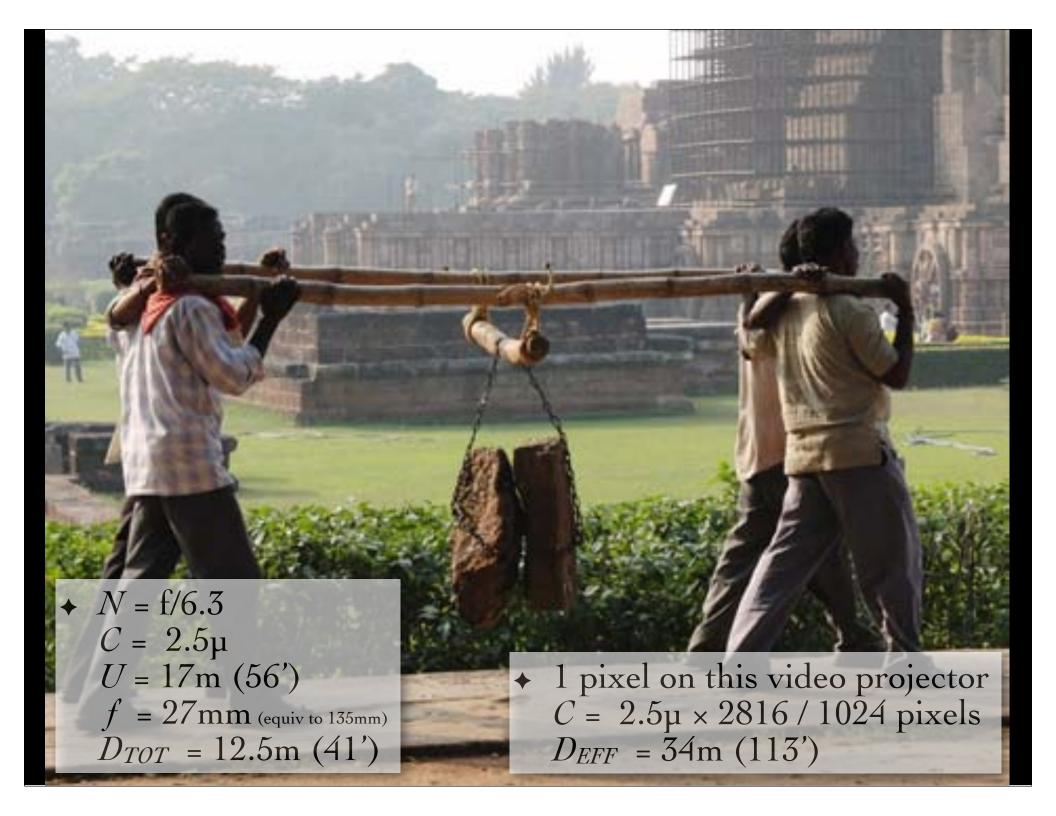
$$D_{TOT} = D_1 + D_2 = \frac{2NCU^2 f^2}{f^4 - N^2 C^2 U^2}$$

♦ $N^2C^2U^2$ can be ignored when conjugate of circle of confusion is small relative to the aperture

$$D_{TOT} \approx \frac{2NCU^2}{f^2}$$

- where
 - N is F-number of lens
 - C is circle of confusion (on image)
 - *U* is distance to in-focus plane (in object space)
 - f is focal length of lens









Canon MP-E 65mm 5:1 macro

N = f/2.8 $C = 6.4\mu$ U = 78mm f = 65mm



(use $N' = (1+M_T)N$ at short conjugates $(M_T=5 \text{ here})$) = f/16 $D_{TOT} = 0.29 \text{mm}!$

(Mikhail Shlemov)

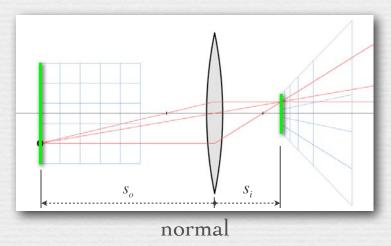
Sidelight: macro lenses

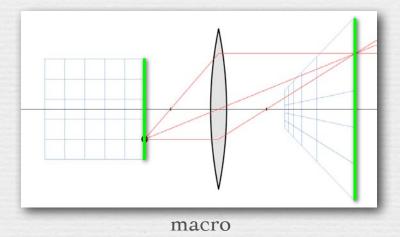
$$\frac{1}{s_o} + \frac{1}{s_i} = \frac{1}{f}$$





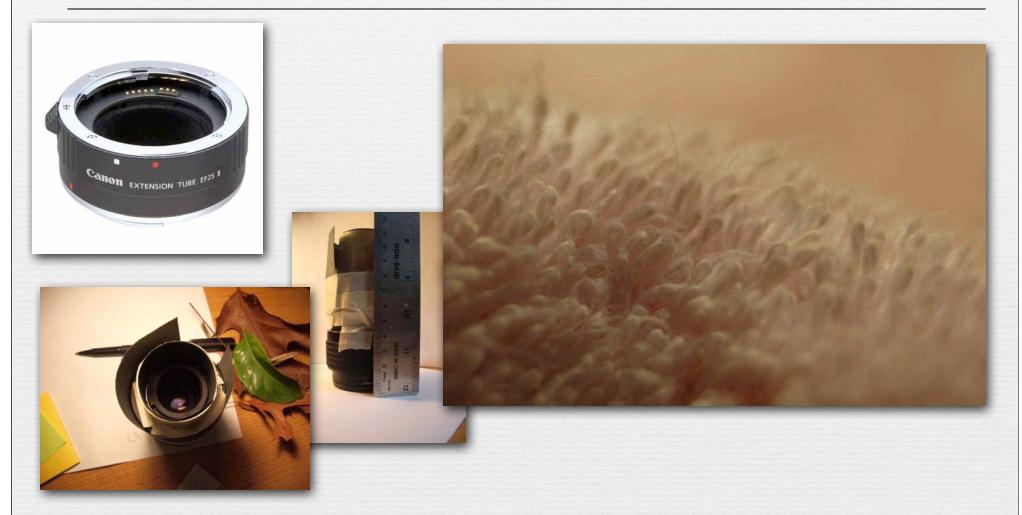
Q. How can the Casio EX-F1 at 73mm and the Canon MP-E 65mm macro, which have similar f's, have such different focusing distances?





- ♦ A. Because macro lenses are built to allow long si
 - this changes s_o , which changes magnification $M_T \triangleq -s_i/s_o$
 - macro lenses are also well corrected for aberrations at short so

Extension tube: fits between camera and lens, converts a normal lens to a macro lens



- → toilet paper tube, black construction paper, masking tape
- → camera hack by Katie Dektar (CS 178, 2009)

Extension tubes versus close-up filters



Students asked about "close-focus" settings on some standard zoom lenses. These indeed allow macro shots to be captured, but such lenses are not well corrected for operating at close object distances, so they will produce mediocrequality macro images.



Canon f = 500mm

- Canon 25mm
- ♦ both allow closer focusing, hence greater magnification
- ◆ both degrade image quality relative to a macro lens
- extension tubes work best with wide-angle lenses;
 close-up filters work best with telephoto lenses
- * extension tubes raise F-number, reducing light
- → need different close-up filter for each lens filter diameter

Extension tubes versus close-up filters versus teleconverters



Canon 25mm



Canon f = 500mm



Nikon 1.4×

- → a teleconverter fits between the camera and lens, like an extension tube
- ◆ they increase f, narrowing FOV & increasing magnification, but they don't change the focusing range
- ♦ like extension tubes, they raise F-number, reducing light, and they are awkward to add or remove
- ♦ See http://www.cambridgeincolour.com/tutorials/macro-extension-tubes-closeup.htm

DoF is linear with F-number

$$D_{TOT} \approx \frac{2NCU^2}{f^2}$$

(FLASH DEMO)

http://graphics.stanford.edu/courses/cs178/applets/dof.html



f/2.8

(juzaphoto.com)



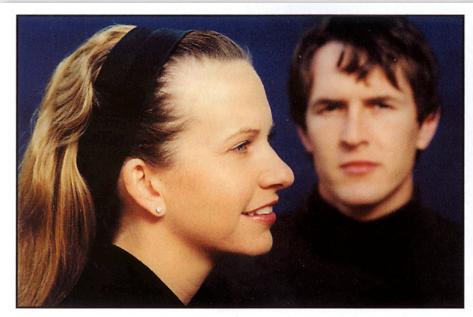
f/32

DoF is quadratic with subject distance

$$D_{TOT} \approx \frac{2NCU^2}{f^2}$$



http://graphics.stanford.edu/courses/ cs178/applets/dof.html



Closer to subject



3 feet



Farther from subject



10 feet

(London)

Hyperfocal distance

the back depth of field

$$D_2 = \frac{NCU^2}{f^2 - NCU}$$

becomes infinite if

$$U \geq \frac{f^2}{NC} \triangleq H$$



♦
$$N = f/6.3$$

 $C = 2.5\mu \times 2816 / 1920 \text{ pixels}$
 $U = 17\text{m } (56')$
 $f = 27\text{mm } (\text{equiv to } 135\text{mm})$
 $D_{TOT} = 18.3\text{m } \text{on HD projector}$
 $H = 31.6\text{m } (104')$

◆ In that case, the front depth of field becomes

$$D_1 = \frac{NCU^2}{f^2 + NCU} = \frac{H}{2}$$

(FLASH DEMO)

http://graphics.stanford.edu/courses/ cs178/applets/dof.html

♦ so if I had focused at 32m, everything from 16m to infinity
would be in focus on a video projector, including the men at 17m

DoF is inverse quadratic with focal length

$$D_{TOT} \approx \frac{2NCU^2}{f^2}$$



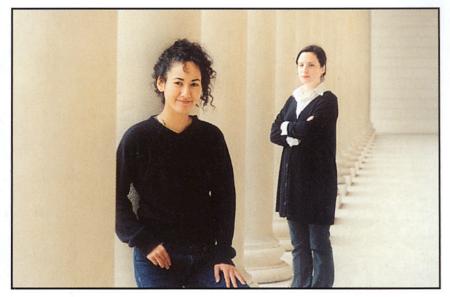
http://graphics.stanford.edu/courses/ cs178/applets/dof.html



Longer focal length



180mm



Shorter focal length



50mm

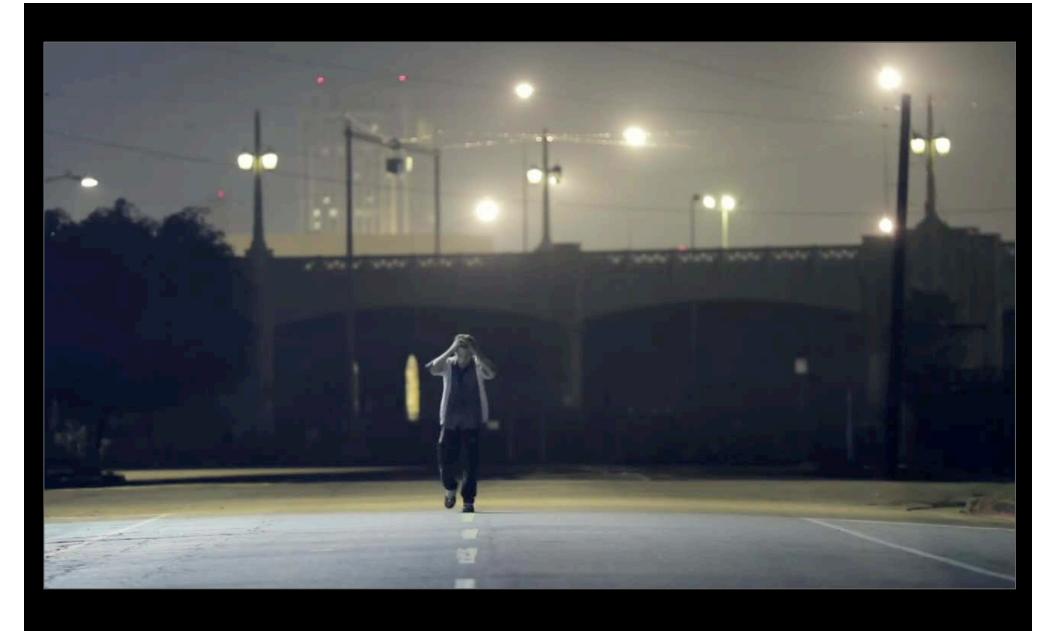
(London)

Q. Does sensor size affect DoF?

$$D_{TOT} \approx \frac{2NCU^2}{f^2}$$

A student asked why full-frame film SLRs have larger film areas than 35mm movie cameras. As I explained in class, in a 35mm movie camera, the film stock is about 24mm wide, and runs vertically in the camera. Within the stock, successive frames are stacked one above the other and are 16mm high by 22mm wide (which leaves room on the side for the audio track). When the same film stock is used in a photographic camera, the film runs horizontally in the camera, rather than vertically, successive frames are placed side-by-side, and are 24mm high by 36mm wide. Hence, the format for still photography really is bigger than for movies, even though they use the same physical film stock.

- ◆ as sensor shrinks, lens focal length f typically shrinks to maintain a comparable field of view
- ◆ as sensor shrinks, pixel size C typically shrinks
 to maintain a comparable number of pixels in the image
- \star thus, depth of field D_{TOT} increases linearly with decreasing sensor size on consumer cameras
- this is why amateur cinematographers are drawn to SLRs
 - their chips are larger than even pro-level video camera chips
 - so they provide unprecedented control over depth of field



Vincent Laforet, Nocturne (2009) Canon 1D Mark IV

DoF and the dolly-zoom

 \star if we zoom in (increase f) and stand further back (increase U) by the same factor

$$D_{TOT} \approx \frac{2NCU^2}{f^2}$$

- depth of field stays the same, but background gets blurrier!
 - useful for macro when you can't get close enough

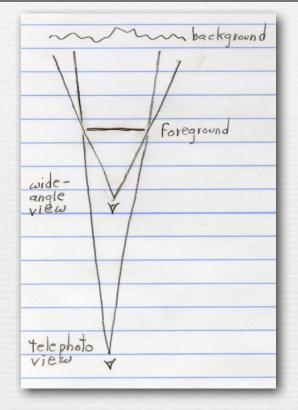


50mm f/4.8



moved back 4× from subject

Macro photography using a telephoto lens (contents of whiteboard)



- changing from a wide-angle lens to a telephoto lens and stepping back,
 you can make a foreground object appear the same size in both lenses
- and both lenses will have the same depth of field on that object
- ♦ but the telephoto sees a smaller part of the background (which it blows up to fill the field of view), so the background will appear blurrier

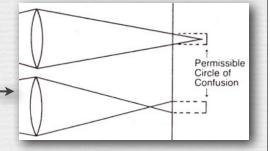
Parting thoughts on DoF: the zen of *bokeh*

Robert Harrington 2003

Canon 85mm prime f/1.8 lens

(wikipedia.org)

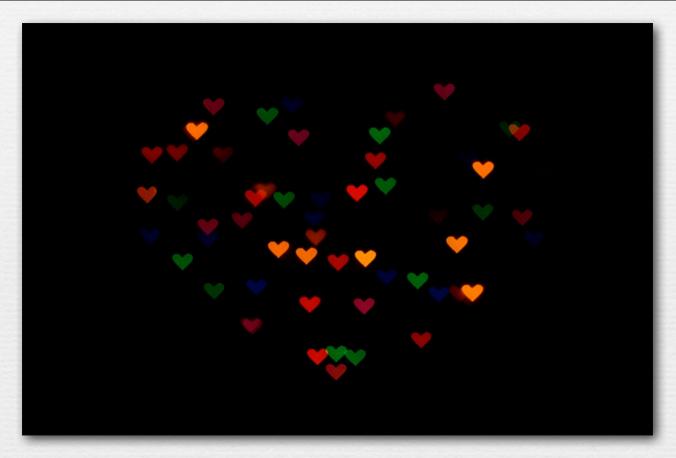
- the appearance of small out-of-focus features in a photograph with shallow depth of field
 - determined by the boundary of the aperture
 - people get religious about it
 - but not every picture with shallow DoF has evident bokeh...





Natasha Gelfand (Canon 100mm f/2.8 prime macro lens)

Games with bokeh



- → picture by Alice Che (CS 178, 2010)
 - heart-shaped mask in front of lens
 - subject was Christmas lights
 - photograph was misfocused and under-exposed

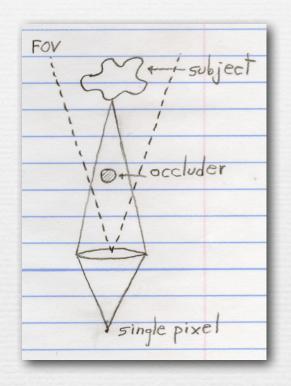
Parting thoughts on DoF: seeing through occlusions



(Fredo Durand)

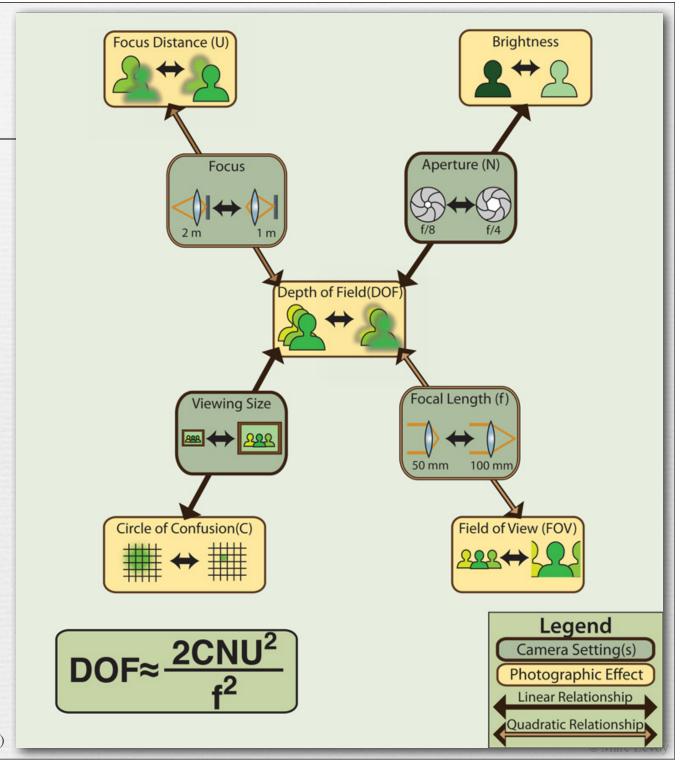
- depth of field is not a convolution of the image
 - i.e. not the same as blurring in Photoshop
 - DoF lets you eliminate occlusions, like a chain-link fence

Seeing through occlusions using a large aperture (contents of whiteboard)



- ♦ for a pixel focused on the subject, some of its rays will strike the occluder, but some will pass to the side of it, if the occluder is small enough
- ♦ the pixel will then be a mixture of the colors of the subject and occluder
- thus, the occluder reduces the contrast of your image of the subject, but it doesn't actually block your view of it

Tradeoffs affecting depth of field



(Eddy Talvala)

Recap

* depth of field (D_{TOT}) is governed by circle of confusion (C), aperture size (N), subject distance (U), and focal length (f)

$$D_{TOT} \approx \frac{2NCU^2}{f^2}$$

- depth of field is linear in some terms and quadratic in others
- if you focus at the hyperfocal distance $H = f^2/NC$, everything from H/2 to infinity will be in focus
- depth of field increases linearly with decreasing sensor size
- useful sidelights
 - bokeh refers to the appearance of small out-of-focus features
 - you can take macro photographs using a telephoto lens
 - depth of field blur is not the same as blurring an image

